Feedback-adjusted carbon prices

Hiroaki Sakamoto*

January 22, 2023

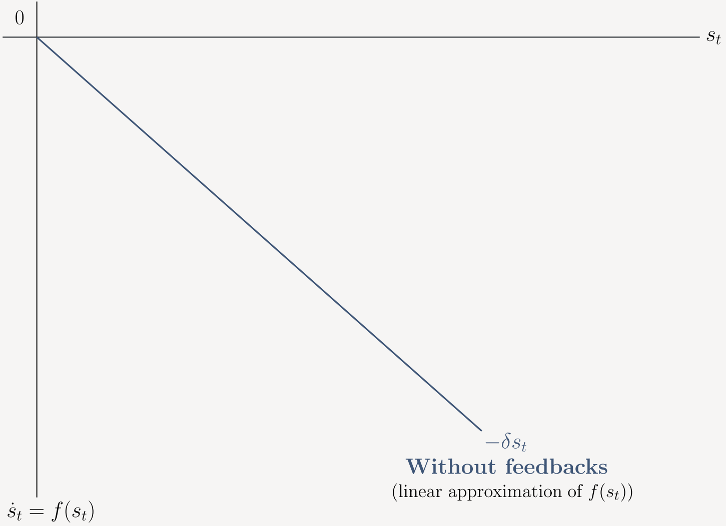
*Graduate School of Economics, Kobe University

Plan of talk

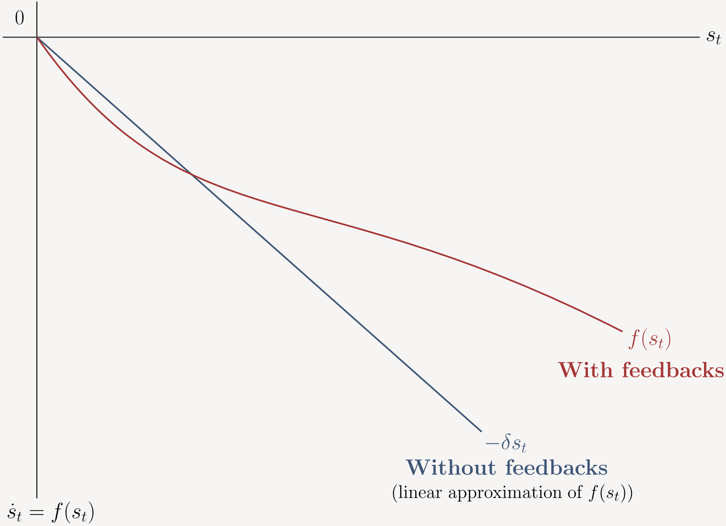
1. Introduction
2. Model
3. Results
4. Conclusions

1 Introduction

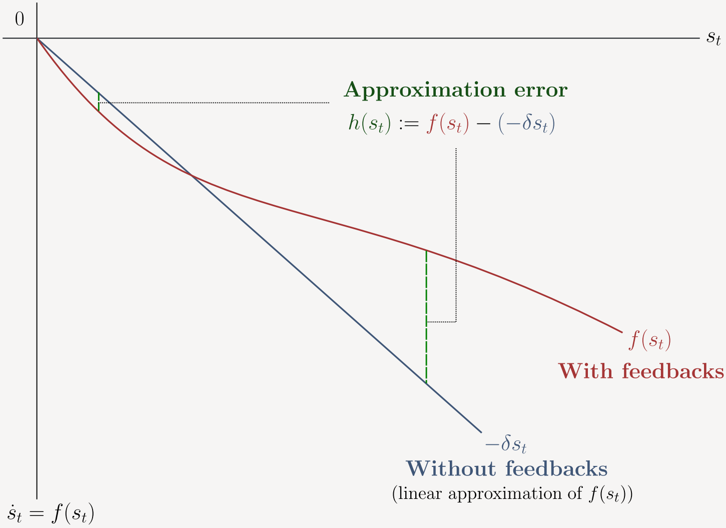
Single-state example ($\dot{s}_t = f(s_t) + x_t$)



Single-state example ($\dot{s}_t = f(s_t) + x_t$)



Single-state example ($\dot{s}_t = f(s_t) + x_t$)



Introduction

Literature

- Numerical models with non-linear feedbacks
  - Hänsel et al. (2020); Dietz et al. (2021b,a); Rennert et al. (2022)
  - results not easy to interpret/communicate
  - limited applicability to theoretical analysis
- Analytical integrated assessment
  - Golosov et al. (2014); Hassler and Krusell (2012); Karp (2017); Gerlagh and Liski (2018a,b); Hillebrand and Hillebrand (2019); Hambel et al. (2021); Iverson and Karp (2021); Traeger (2023)
  - highly tractable
  - under the assumption of linear-in-state structure

Introduction

This paper

- Analytical integrated assessment with feedbacks
  - no longer linear in state variables
  - nevertheless permits a closed-form solution
- General analytical insights
  - social cost of carbon as a function of climate state
  - characterization of feedback premium
- Impact of future technological changes
  - irrelevant in linear models
  - materialize through non-linear channels

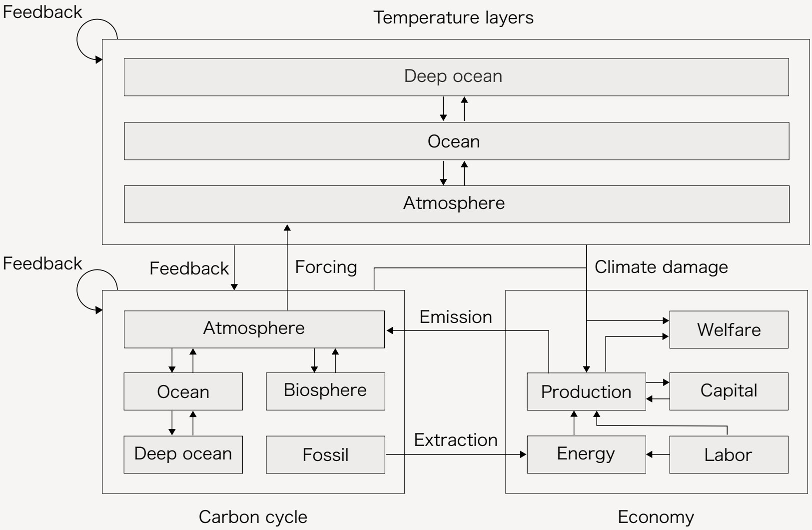
2 Model

Big picture

Set-up

Social cost of carbon

Model — Big picture



Preference

Starting from time t, people's preference is

$$W_t := \int_t^{\infty} e^{-\rho(\tau - t)} \ln(c_\tau) d\tau, \tag{1}$$

where

- c_τ is consumption at period τ,
- ρ is the rate of time preference.

Logarithmic utility (just as in the other studies) is the key.

Model — Set-up

Economy

Based on Rebelo (1991), economy is described as follows:

$$c_t = \Omega_t(\boldsymbol{s}_t)(k_t^y)^{\alpha_k}(l_t^y)^{\alpha_l} z_t^{1-\alpha_k-\alpha_l}, \quad \alpha_k, \alpha_l, \alpha_k + \alpha_l \in (0,1), \tag{2}$$

$$\dot{k}_t = -\delta_k k_t + \Omega_t^k (k_t - k_t^y), \quad \delta_k > 0, \Omega_t^k > 0. \tag{3}$$

$$z_t = \Omega_t^z {x_t}^{\alpha_x} (l_t - l_t^y)^{1-\alpha_x}, \quad \alpha_x \in (0,1), \Omega_t^z > 0. \tag{4}$$

where

- k_t capital, l_t labor, z_t energy, x_t carbon input (fossil fuel),
- $\boldsymbol{s}_t = (\boldsymbol{M}_t^\top, \boldsymbol{T}_t^\top)^\top$ the vector of climate related state variables,
- $\boldsymbol{M}_t = (M_{1,t}, \ldots, M_{I,t})^\top$ carbon stocks in different reservoirs and $\boldsymbol{T}_t = (T_{1,t}, \ldots, T_{J,t})^\top$ temperatures in different layers,
- climate impact through $\Omega_t(\boldsymbol{s}_t)$

Model — Set-up

Climate system

Modeled by the following $I + J$ equations of motion:

$$\dot{\boldsymbol{s}}_t = \boldsymbol{f}(\boldsymbol{s}_t) + \boldsymbol{e}_1 x_t, \tag{5}$$

where

- $\boldsymbol{e}_1 := (1, 0, \ldots, 0)^\top$
- $\boldsymbol{f} : \mathbb{R}^{I+J} \to \mathbb{R}^{I+J}$ governs the time evolution of $\boldsymbol{s}_t$:
  - typically linear, i.e., $\boldsymbol{f}(\boldsymbol{s}) = \boldsymbol{\Phi}\boldsymbol{s}$ for some matrix $\boldsymbol{\Phi}$
  - in this paper generalized to allow for non-linear feedback effects

Model — Set-up

Lemma 1 (Separability)

Combining (1)–(4), *one may rewrite the welfare as*

$$W_t = \bar{w}_t + \frac{\alpha_k}{\rho}\ln(k_t) + \underbrace{\int_t^{\infty} e^{-\rho(\tau-t)} u(x_\tau, \boldsymbol{s}_\tau, \tau) d\tau}_{\text{'reduced form'}}, \qquad (6)$$

where $u(x, \boldsymbol{s}, t) := \gamma \ln(x) + \ln(\Omega_t(\boldsymbol{s}))$ *with* $\gamma := (1 - \alpha_k - \alpha_l)\alpha_x$, *and* $\bar{w}_t$ *a function of* $(l^y_\tau / l_\tau, k^y_\tau / k_\tau)_{\tau \geq t}$.

Observe:

- climate and other variables are additively separable,
- the last term in (6) is a 'reduced form' of the model.

Optimal social cost of carbon: definition

The optimal path of carbon consumption solves

$$V(\boldsymbol{s}, t) := \max_{(x_\tau)_{\tau=t}^{\infty}} \int_t^{\infty} e^{-\rho(\tau-t)} u(x_\tau, \boldsymbol{s}_\tau, \tau) d\tau \tag{7}$$

subject to climate dynamics (5).

Definition 2 (Social cost of carbon: SCC)

The social cost of carbon at time t is

$$\mathrm{SCC}_t = -\frac{\partial V(\boldsymbol{s}_t, t)}{\partial M_{1,t}} \Big/ \frac{d \ln(c_t)}{dc_t}.$$

Model — Social cost of carbon

Lemma 3 (SCC: a general expression)

If $x_t = \sigma(\boldsymbol{s}_t, t)$ is the optimal policy for carbon emission, the social cost of carbon at time t is given by

$$\mathrm{SCC}_t = \frac{\gamma}{\sigma(\boldsymbol{s}_t, t)} c_t. \tag{8}$$

- SCC is proportional to the current consumption level c_t,
- regardless of how damage $\Omega_t(\boldsymbol{s})$ and climate $\boldsymbol{f}(\boldsymbol{s})$ specified.

3 Results

Linear model

Non-linear model

Quantification

Results — Linear model

Assumption 1 (Exponential damage function)

The damage function $\Omega_t(\boldsymbol{s})$ is an exponential function so that the reduced-form damage function is linear:

$$\ln\left(\Omega_t(\boldsymbol{s})\right) = \bar{\omega}_t + \boldsymbol{\zeta}\boldsymbol{s}, \quad \textit{where} \quad \boldsymbol{\zeta} = (\zeta_1, \ldots, \zeta_J). \tag{9}$$

Results — Linear model

Linear carbon cycle

$$\underbrace{\begin{pmatrix} \dot{M}_{1,t} \\ \dot{M}_{2,t} \\ \vdots \\ \dot{M}_{I,t} \end{pmatrix}}_{=:\dot{\boldsymbol{M}}_t} = \underbrace{\begin{pmatrix} \delta_{1,1} & \delta_{1,2} & \dots & \delta_{1,I} \\ \delta_{2,1} & \delta_{2,2} & \dots & \delta_{2,I} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{I,1} & \delta_{I,2} & \dots & \delta_{I,I} \end{pmatrix}}_{=:\boldsymbol{\Delta}} \underbrace{\begin{pmatrix} M_{1,t} \\ M_{2,t} \\ \vdots \\ M_{I,t} \end{pmatrix}}_{=:\boldsymbol{M}_t} + \boldsymbol{e}_1 x_t, \qquad (10)$$

where

- $\sum_{i'=1}^{I} \delta_{i',i} = 0$ for all i (i.e., carbon never leaks out).
- matrix $\boldsymbol{\Delta}$ is independent of $\boldsymbol{s}_t = (\boldsymbol{M}_t^\top, \boldsymbol{T}_t^\top)^\top$.

Linear temperature dynamics

$$\underbrace{\begin{pmatrix} \dot{T}_{1,t} \\ \dot{T}_{2,t} \\ \vdots \\ \dot{T}_{J,t} \end{pmatrix}}_{=:\dot{\boldsymbol{T}}_t} = \underbrace{\begin{pmatrix} \theta_{1,1} & \theta_{1,2} & \dots & \theta_{1,J} \\ \theta_{2,1} & \theta_{2,2} & \dots & \theta_{2,J} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{J,1} & \theta_{J,2} & \dots & \theta_{J,J} \end{pmatrix}}_{=:\boldsymbol{\Theta}} \underbrace{\begin{pmatrix} T_{1,t} \\ T_{2,t} \\ \vdots \\ T_{J,t} \end{pmatrix}}_{=:\boldsymbol{T}_t} + \varphi \boldsymbol{e}_1 M_{1,t}, \qquad (11)$$

where

- $T_{j,t}$ is an exponential transformation of raw temperature (to capture logarithmic carbon forcing) (Traeger, 2023),
- but again, matrix $\boldsymbol{\Theta}$ is independent of $\boldsymbol{s}_t = (\boldsymbol{M}_t^\top, \boldsymbol{T}_t^\top)^\top$.

Results — Linear model

Assumption 2 (Linear climate system)

The climate system is linear in (transformed) state variables:

$$\boldsymbol{f}(\boldsymbol{s}) = \boldsymbol{\Phi}\boldsymbol{s}, \tag{12}$$

where

$$\boldsymbol{\Phi} := \begin{pmatrix} \delta_{1,1} & \delta_{1,2} & \dots & \delta_{1,I} & 0 & 0 & \dots & 0 \\ \delta_{2,1} & \delta_{2,2} & \dots & \delta_{2,I} & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \delta_{I,1} & \delta_{I,2} & \dots & \delta_{I,I} & 0 & 0 & \dots & 0 \\ \varphi & 0 & \dots & 0 & \theta_{1,1} & \theta_{1,2} & \dots & \theta_{1,J} \\ 0 & 0 & \dots & 0 & \theta_{2,1} & \theta_{2,2} & \dots & \theta_{2,J} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & \theta_{J,1} & \theta_{J,2} & \dots & \theta_{J,J} \end{pmatrix}. \tag{13}$$

Proposition 1 (Solution of linear model)

Under Assumptions 1 and 2, the value function is

$$V(\boldsymbol{s}, t) = \bar{\upsilon}_t - \boldsymbol{\xi}\boldsymbol{s}, \tag{14}$$

where the shadow cost vector, $\boldsymbol{\xi} = (\xi_1, \ldots, \xi_{I+J})$*, is*

$$\boldsymbol{\xi} := \zeta \left(\rho \boldsymbol{I} - \boldsymbol{\Phi}\right)^{-1}. \tag{15}$$

Note that the shadow cost $\boldsymbol{\xi}$:

- reflects the raw damage coefficient ζ,
- adjusted by discounting ρ and climate dynamics $\boldsymbol{\Phi}$ ($\boldsymbol{\Delta}$ and $\boldsymbol{\Theta}$)

Results — Linear model

Proposition 2 (SCC of linear model)

Under Assumptions 1 and 2, the optimal SCC is

$$\mathrm{SCC}_t = \xi_1 c_t. \tag{16}$$

This result, well known since Golosov et al. (2014), shows:

- SCC is independent of climate states
- delayed mitigation does not require any policy change

Results — Non-linear model

The idea

- Linear model has approximation error, $\boldsymbol{f}(\boldsymbol{s}) - \Phi\boldsymbol{s} =: \boldsymbol{h}$
- The error, $\boldsymbol{h}(\boldsymbol{s})$ in general, may be reasonably picked up by a function of cumulative emission $M := \sum_i M_{i,t}$:

$$\boldsymbol{f}(\boldsymbol{s}) - \Phi\boldsymbol{s} \approx \boldsymbol{h}(M)$$

  for some function $\boldsymbol{h} : \mathbb{R} \to \mathbb{R}^{I+J}$
- Consistent with FAIR model of Millar et al. (2017)

Results — Non-linear model

Assumption 3 (Non-linear climate system)

The climate states are governed by

$$\boldsymbol{f}(\boldsymbol{s}) = \Phi \boldsymbol{s} + \boldsymbol{h}(M) \tag{17}$$

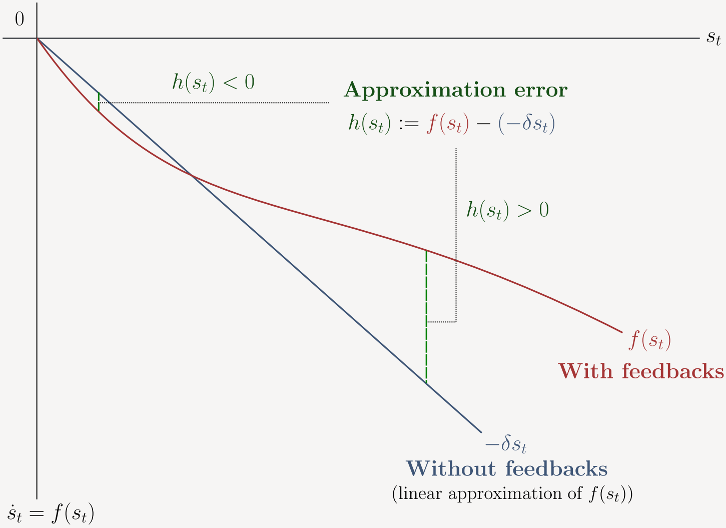
where $\boldsymbol{h}(M) = (h_1(M), \ldots, h_{I+J}(M))^\top$ *is an arbitrary function that satisfies* $\sum_{i=1}^{I} h_i(M) = 0$ *for all* M *and* $\boldsymbol{h}(0) = \mathbf{0}$.

Define the damage-weighted linear approximation error by

$$h_\xi(M) := \boldsymbol{\xi} \boldsymbol{h}(M). \tag{18}$$

- $h_\xi(M) < 0$: linear model underestimates the capability of climate system to handle carbon emissions
- $h_\xi(M) > 0$: linear model rather overestimates the carbon- and heat-absorbing capacity of the system

Single-state example ($\dot{s}_t = f(s_t) + x_t$)



Results — Non-linear model

Proposition 3 (Solution of non-linear model)

Under Assumptions 1 and 3, the value function is

$$V(\boldsymbol{s}, t) = \bar{v}_t - \boldsymbol{\xi}\boldsymbol{s} - \Psi(M), \tag{19}$$

where $\Psi(M)$ *is*

$$\Psi(M) := \frac{\gamma}{\rho} \ln \left(\frac{e^{-\rho \frac{\xi_1}{\gamma} M}}{\rho \frac{\xi_1}{\gamma} \int_M^\infty e^{-\rho \frac{\xi_1}{\gamma} m - \frac{1}{\gamma} h_\xi(m)} dm} \right).$$

- Marginal utility damage is no longer constant
- $\Psi(M)$ does not only depend on the current feedback effects, but also reflects all possible feedbacks that could follow

Results — Non-linear model

Proposition 4 (SCC of non-linear model)

Under Assumptions 1 and 3, the feedback-adjusted optimal SCC is

$$\text{SCC}_t = \psi(M)\xi_1 c_t, \tag{20}$$

where

$$\psi(M) := \frac{e^{-\rho\frac{\xi_1}{\gamma}M - \frac{1}{\gamma}h_\xi(M)}}{\rho\frac{\xi_1}{\gamma}\int_M^\infty e^{-\rho\frac{\xi_1}{\gamma}m - \frac{1}{\gamma}h_\xi(m)}dm}. \tag{21}$$

Note that

- with no feedback, SCC is given by $\xi_1 c_t$;
- scaling factor, $\psi(M)$, captures the feedback premium

Results — Non-linear model

Proposition 5 (Characterization of feedback premium)

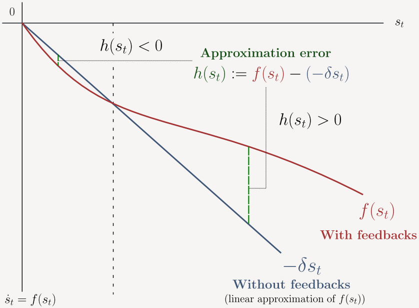
For each level M of cumulative emission,

$$\psi(M) \geq 1 \iff \int_M^\infty h'_\xi(m) e^{-\rho\frac{\xi_1}{\gamma}m - \frac{1}{\gamma}h_\xi(m)} dm \geq 0 \qquad (22)$$

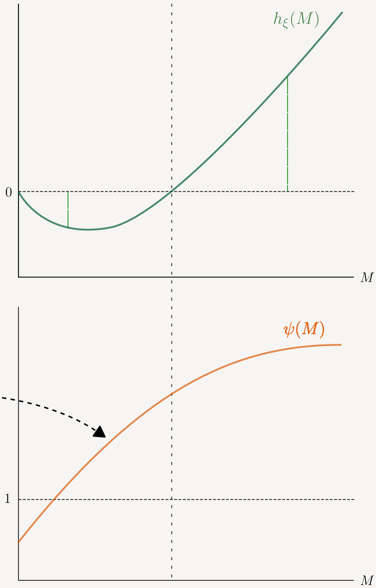
and

$$\psi'(M) \geq 0 \iff \int_M^\infty h''_\xi(m) \frac{e^{-\rho\frac{\xi_1}{\gamma}m - \frac{1}{\gamma}h_\xi(m)}}{\left(\rho\frac{\xi_1}{\gamma} + \frac{1}{\gamma}h'_\xi(m)\right)^2} dm \geq 0. \qquad (23)$$

Results — Non-linear model



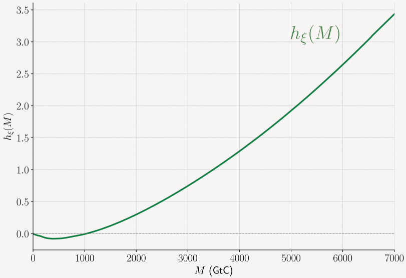
Positive feedback premium even when the approximation error is negative

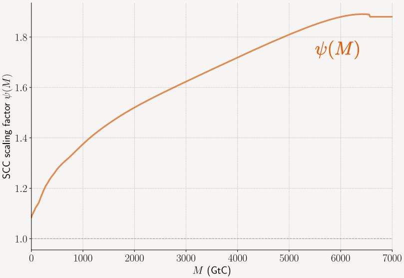


Calibration

- $I = 4$ for carbon cycle and calibrate $\boldsymbol{\Delta}$ based on Joos et al. (2013) and $\boldsymbol{h}(M)$ based on Millar et al. (2017)
- $J = 3$ for temperature and calibrate $\boldsymbol{\Theta}$ to replicate Geoffroy et al. (2013)'s simulations
- Damage coefficient ζ is based on DICE
- time preference rate is $\rho = 0.015$
- $\gamma = 0.018$ is the income share of fossil fuel industry

Approximation error $h_\xi(M)$ and feedback premium $\psi(M)$

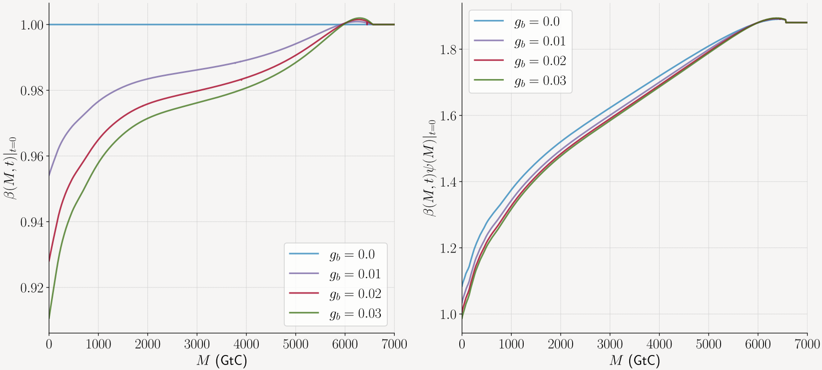




- at the current level ($M = 670$), the feedback premium is 31%
- in spite of the fact that the linear climate model appears to be a good approximation for now

Results — Quantification

Impact of future decarbonization on today's carbon price



- expected future decarbonization lowers today's SCC
- but quantitatively insignificant (merely by a few percent)

4 Conclusions

Conclusions

Key findings

- Feedback effects increase the optimal carbon price way before those effects physically kick in
- Actions needed when the climate system is still in good shape
- Delayed mitigation implies a larger feedback premium later
- Expected future decarbonization lowers the carbon price today, but quantitatively the impact is insignificant

References I

Simon Dietz, James Rising, Thomas Stoerk, and Gernot Wagner. Economic impacts of tipping points in the climate system. **Proceedings of the National Academy of Sciences of the United States of America**, 118(34), 2021a.

Simon Dietz, Frederick van der Ploeg, Armon Rezai, and Frank Venmans. Are economists getting climate dynamics right and does it matter? **Journal of the Association of Environmental and Resource Economists**, 8(5):895–921, 2021b.

O Geoffroy, D Saint-Martin, D J L Olivié, A Voldoire, G Bellon, and S Tytéca. Transient climate response in a Two-Layer Energy-Balance model. part i: Analytical solution and parameter calibration using CMIP5 AOGCM experiments. **Journal of Climate**, 26(6):1841–1857, 2013.

Reyer Gerlagh and Matti Liski. Consistent climate policies. **Journal of the European Economic Association**, 16(1):1–44, 2018a.

References II

Reyer Gerlagh and Matti Liski. Carbon prices for the next hundred years. **Economic Journal**, 128(609):728–757, 2018b.

Mikhail Golosov, John Hassler, Per Krusell, and Aleh Tsyvinski. Optimal taxes on fossil fuel in general equilibrium. **Econometrica**, 82(1):41–88, 2014.

Christoph Hambel, Holger Kraft, and Eduardo Schwartz. The social cost of carbon in a non-cooperative world. **Journal of International Economics**, 131:103490, 2021.

Martin C Hänsel, Moritz A Drupp, Daniel J A Johansson, Frikk Nesje, Christian Azar, Mark C Freeman, Ben Groom, and Thomas Sterner. Climate economics support for the UN climate targets. **Nature Climate Change**, 10(8):781–789, 2020.

John Hassler and Per Krusell. Economics and climate change: integrated assessment in a multi-region world. **Journal of the European Economic Association**, 10(5):974–1000, 2012.

References III

Elmar Hillebrand and Marten Hillebrand. Optimal climate policies in a dynamic multi-country equilibrium model. **Journal of Economic Theory**, 179:200–239, 2019.

Terrence Iverson and Larry Karp. Carbon taxes and climate commitment with non-constant time preference. **Review of Economic Studies**, 88(2):764–799, 2021.

Fortunat Joos, Raphael Roth, Jan S Fuglestvedt, Glen P Peters, Ian G Enting, W von Bloh, Victor Brovkin, Eleanor J Burke, Michael Eby, Neil R Edwards, and Others. Carbon dioxide and climate impulse response functions for the computation of greenhouse gas metrics: a multi-model analysis. **Atmospheric Chemistry and Physics**, 13(5):2793–2825, 2013.

Larry S. Karp. Provision of a public good with multiple dynasties. **Economic Journal**, 127(607):2641–2664, 2017.

Richard J. Millar, Zebedee R. Nicholls, Pierre Friedlingstein, and Myles R. Allen. A modified impulse-response representation of the global near-surface air temperature and atmospheric concentration response to carbon dioxide emissions. **Atmospheric Chemistry and Physics**, 17:7213–7228, 2017.

Sergio Rebelo. Long-run policy analysis and long-run growth. **Journal of Political Economy**, 99(3):500–521, 1991.

Kevin Rennert, Frank Errickson, Brian C Prest, Lisa Rennels, Richard G Newell, William Pizer, Cora Kingdon, Jordan Wingenroth, Roger Cooke, Bryan Parthum, David Smith, Kevin Cromar, Delavane Diaz, Frances C Moore, Ulrich K Müller, Richard J Plevin, Adrian E Raftery, Hana Ševčíková, Hannah Sheets, James H Stock, Tammy Tan, Mark Watson, Tony E Wong, and David Anthoff. Comprehensive evidence implies a higher social cost of CO2. **Nature**, 610(7933):687–692, 2022.

References V

Christian P. Traeger. Ace—analytic climate economy. **American Economic Journal: Economic Policy**, forthcoming, 2023.