Intermediate public economics 6
Public goods

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Contents

Private goods, public goods

Private goods (as opposed to public goods)

- Most of the goods have the following properties:
  - rivalry: once consumed, nobody else can consume the same good (simultaneously, at least)
  - excludability: people who do not pay the fair cost can be prevented from enjoying the benefit of the good

Public goods

- Public goods are a particular type of goods which have both (or part) of the following properties:
  - non-rivalry: consumption by one person does not affect the consumption opportunities of the others
  - non-excludability: once provided, no one can be prevented from enjoying the benefits

Real-world examples

Pure public goods (almost, I should say)

- Common examples of public goods include:
  - national defense services
  - lighthouse and street lighting
  - environmental services (e.g., clean air)

Impure public goods

- Goods in between private and pure public goods:

  a) broadcasting services (excludable),
  b) fishery stock in the high seas (rivalrous),
  c) national parks and roads (partially rivalrous, excludable at some cost),

  where goods of type a) is called club goods while b) is called common-pool (or -property) resources

Why public goods?

Another type of market failure

- You can enjoy the benefit of public goods without purchasing them (once purchased by somebody else)
- Who in the world would pay for public goods then?
- Incentive to supply public goods is inevitably weak since they are not sufficiently paid for

→ makes it difficult for public goods to be traded in market

Free-riding and inefficiency

- This is what we call the free-riding problem, a consequence of non-rivalry and non-excludability
- Due to free-riding, public goods are likely to be undersupplied relative to the efficient level
- Government, as a result, should play a role

Simple example

Public broadcasting service

- Two consumers, A and B
- Strategies: P (pay fee) or N (not pay fee)
- High-quality TV program only supplied when both pay
- Otherwise low-budget poor-quality program supplied

Payoff matrix

- Consider the payoff matrix listed below
- Assume $2c > g > c > 0$ and find the Nash equilibrium

		Consumer B	
		P	N
Consumer A	P	$g - c\, ,\, g - c$	$g/2 - c\, ,\, g/2$
	N	$g/2\, ,\, g/2 - c$	$0\, ,\, 0$

Modelling public goods

Preference

- $n \geq 2$ consumers in the economy
- Utility function is $U^i(g, x_i)$, where x_i is consumption of private good whereas g is consumption of public good
- i has endowment $\bar{x}_i > 0$ of private good (numéraire)
- $\bar{x}_i$ can be used for contribution $c_i \in [0, \bar{x}_i]$ for public good:

$$x_i + c_i = \bar{x}_i \quad (1)$$

Technology

- Consumers know that public good is produced by

$$g = G(\textstyle\sum_{i=1}^{n} c_i) \quad (2)$$

- Cost of producing g units of public goods is hence

$$C(g) := G^{-1}(g) \quad (3)$$

Characterizing efficient allocation

Theorem (Samuelson, 1954)

- If an allocation $(g^*, x_1^*, \dots, x_n^*)$ is Pareto efficient, then it must be the case that

$$\sum_{i=1}^{n} \frac{U_g^i(g^*, x_i^*)}{U_x^i(g^*, x_i^*)} = C'(g^*), \tag{4}$$

  which is called the Samuelson condition
- In other words, sum of MRS should be equal to MRT

Proof (sketch)

- Suppose that (4) does not hold, say LHS $>$ RHS
- Then, for sufficiently small $\varepsilon \in \mathbb{R}_{++}$, there must exist $(\delta_1, \dots, \delta_n) \in \mathbb{R}_{++}^n$ such that reallocation to $(g^* + \varepsilon, x_1^* - \delta_1, \dots, x_n^* - \delta_n)$ is feasible and Pareto-improving

Nash equilibrium

Public goods provision game

- Public goods provision involves strategic interaction among consumers
- You would have little incentive to make a contribution c_i if everybody else's contribution c_{-i} is sufficient
- Considered to be a game

Nash equilibrium

- Profile $(c_i^N)_{i=1}^n$ of contribution is a Nash equilibrium if

$$u^i(c_i^N, c_{-i}^N) \geq u^i(c_i, c_{-i}^N) \quad \forall c_i \in [0, \bar{x}_i] \tag{5}$$

for all $i \in \{1, 2, \ldots, n\}$, where

$$u^i(c_i, c_{-i}) := U^i(G(c_i + \textstyle\sum_{j \neq i} c_i), \bar{x}_i - c_i) \tag{6}$$

Characterizing Nash equilibrium

First-order condition

- If $(c_i^N)_{i=1}^n$ is an (interior) Nash equilibrium, then it must be the case that

$$\frac{U_g^i(g^N, x_i^N)}{U_x^i(g^N, x_i^N)} = C'(g^N), \tag{7}$$

where

$$g^N := G(\textstyle\sum_i c_i^N) \text{ and } x_i^N := \bar{x}_i - c_i^N \tag{8}$$

Inefficiency

- Notice that (9) implies

$$\sum_{i=1}^{n} \frac{U_g^i(g^N, x_i^N)}{U_x^i(g^N, x_i^N)} > C'(g^N), \tag{9}$$

violating the Samuelson condition (undersupplied)

Example

Setup and Nash equilibrium

- $U^i(g, x_i) := \gamma_i \ln(g) + \ln(x_i)$ for some $\gamma_i > 0$
- $G(c) := c^{1/2}$ and hence $C(g) := g^2$
- At eqm,

$$\gamma_i \frac{\bar{x}_i - c_i^N}{g^N} = 2g^N \tag{10}$$

- For sufficiently small $\varepsilon \in \mathbb{R}_{++}$, define $\delta_i \in \mathbb{R}_{++}$ by

$$\delta_i := \left(\gamma_i \frac{\bar{x}_i - c_i^N}{g^N} - \frac{1}{n} \left(\sum_j \gamma_j \frac{\bar{x}_j - c_j^N}{g^N} - 2g^N \right) \right) \varepsilon \tag{11}$$

- Then $(g^N + \varepsilon, x_1^N - \delta_1, \ldots, x_n^N - \delta_n)$ Pareto-dominates the eqm allocation $(g^N, x_1^N, \ldots, x_n^N)$

Another example (quasi-linear utility)

Setup and Nash equilibrium

- $U^i(g, x_i) := \gamma_i \ln(g) + x_i$ for some $\gamma_i > 0$
- Assume $\gamma_1 > \gamma_j$ for all $j \neq 1$
- $G(c) := c^{1/2}$ and hence $C(g) := g^2$
- In this case, eqm involves corner solution:

$$c_j^N = 0 \quad \text{for all } j \neq 1 \tag{12}$$

  and

$$c_1^N = 2^{-1}\gamma_1, \quad g^N = G(c_1^N) = (2^{-1}\gamma_1)^{1/2} \tag{13}$$

- Only consumer 1, who most highly values the public good, makes a contribution and everybody else completely free-rides!
- Inefficiency: $g^N < g^* := (2^{-1}\sum_j \gamma_j)^{1/2}$

Lindahl mechanism: the idea

Personalized tax system

- Reasons why public goods are undersupplied:
  - strategic incentive
  - marginal value of public goods varies across consumers, which is not taken into account
- Remove the strategic interaction (among consumers) by transforming the game into market-like setting
- Personalize the price so that everybody pays a fair share of the cost

Stark contrast to private goods

- Private goods: shared price $\times$ personalized quantity
- Public goods: personalized price $\times$ shared quantity

Lindahl mechanism: procedure

Procedure

- First government announces a personalized tax-transfer system $(\tau_i, T_i)_{i=1}^n$
- Consumer i's 'demand' for g is determined by

$$(g_i^d(\tau_i, T_i), x_i^d(\tau_i, T_i)) \in \operatorname{argmax} U^i(g, x_i) \text{ s.t. } x_i + \tau_i g = \bar{x}_i + T_i$$

- Given the feedback $(g_i^d(\tau_i, T_i))_{i=1}^n$ from consumers, government adjusts $(\tau_i^*, T_i^*)_{i=1}^n$ in such a way that

$$g_i^d(\tau_i^*, T_i^*) = g_j^d(\tau_j^*, T_j^*) =: g^* \text{ for all } i, j \tag{14}$$

and

$$\textstyle\sum_i \tau_i^* = C'(g^*), \quad \sum_i T_i^* + C(g^*) = \sum_i \tau_i^* g^* \tag{15}$$

- This tax-transfer system is called Lindahl mechanism

Lindahl equilibrium (quasi-linear utility)

Computing Lindahl tax rate

- $U^i(g, x_i) := \gamma_i \ln(g) + x_i$ for some $\gamma_i > 0$
- $G(c) := c^{1/2}$ and hence $C(g) := g^2$
- Demand function $g_i^d(\tau_i)$ is

$$g_i^d(\tau_i) = \gamma_i / \tau_i \tag{16}$$

- Set τ_i^* and T_i^* by

$$\tau_i^* := \gamma_i (2^{-1} \textstyle\sum_j \gamma_j)^{-1/2}, \quad T_i^* := \frac{1}{2n} (\textstyle\sum_j \gamma_j) \tag{17}$$

  so that $g_i^d(\tau_i^*) = (2^{-1} \sum_j \gamma_j)^{1/2} =: g^*$ for all i
- Efficiency restored b/c

$$\sum_i MRS(g^*, x_i^*) = \sum_i \gamma_i / g^* = 2g^* = MRT(g^*) \tag{18}$$

Practical relevance

Vulnerable to ‘cheating’

- In the Lindahl mechanism, we essentially ask consumers to report their preference for public goods
- In principle, consumers can tell a lie, misrepresenting her demand for public goods
- Mechanism only works if everybody truthfully reveals their preference

Any incentive to tell a lie?

- Suppose that consumers know how the Lindahl mechanism works
- Then they will realize that they can be (individually) better off by understating their demand for public goods
- Incentive to report $\tilde{g}_i^d(\tau_i) = \tilde{\gamma}_i / \tau_i$ for some $\tilde{\gamma}_i < \gamma_i$

Alternative to Lindahl mechanism

Mechanism design

- In general, a mechanism is said to be demand-revealing if nobody has an incentive to misrepresent their demand
- Lindahl mechanism is, by design, not demand-revealing
- Is it possible to design a mechanism that is demand-revealing?

Vickrey-Clarke-Groves mechanism

- One example of demand-revealing mechanisms is Vickrey-Clarke-Groves (VCG) mechanism
- Under this mechanism, everybody truthfully reveals their preference even if they are not required to do so
- But this is only achieved at the cost of efficiency

VCG mechanism: procedure

Quasi-linear utility

- Consumers' preference is represented by

$$U^i(g, x_i) := v_i(g) + x_i \tag{19}$$

- Here $v_i(g)$ is the true valuation of public goods, which is unknown to government
- Special case is $v_i(g) := \gamma_i \ln(g)$

Procedure

- Government first asks consumers to reveal $v_i : \mathbb{R}_+ \to \mathbb{R}$
- Consumers report $\tilde{v}_i : \mathbb{R}_+ \to \mathbb{R}$ (can be different from v_i)
- Given the feedback $\tilde{v} := (\tilde{v}_i)_{i=1}^N$, government then decides the level $g(\tilde{v})$ of public goods as well as the personalized lump-sum tax $c_i(\tilde{v})$

VCG mechanism: design

Designing the mechanism

- Prior to asking consumers to reveal their preference, government announces that $g(\tilde{v})$ is determined by

$$g(\tilde{v}) \in \operatorname*{argmax}_{g} \left(\sum_{i=1}^{N} \tilde{v}_i(g) - C(g) \right) \qquad (20)$$

  and $c_i(\tilde{v})$ is determined by

$$c_i(\tilde{v}) := \bar{t}_i(\tilde{v}) - \left(\sum_{j \neq i} \tilde{v}_j(g(\tilde{v})) - C(g(\tilde{v})) \right) \qquad (21)$$

  for some $\bar{t}_i(\tilde{v}) \in \mathbb{R}$ (which can depend on $\tilde{v} := (\tilde{v}_i)_{i=1}^{N}$)
- Consumer i's utility is then

$$u^i(\tilde{v}_i, \tilde{v}_{-i}) := v_i(g(\tilde{v})) + \bar{x}_i - c_i(\tilde{v}) \qquad (22)$$

VCG mechanism: Nash equilibrium

Truth-telling is a dominant strategy

- Suppose that everybody else truthfully reveals their preference, i.e.,

$$\tilde{v}_j = v_j \quad \forall j \neq i \tag{23}$$

- Then, consumer i's utility is

$$\begin{aligned} u^i(\tilde{v}_i, v_{-i}) &= v_i(g(\tilde{v}_i, v_{-i})) + \bar{x}_i - c_i(\tilde{v}_i, v_{-i}) \\ &\propto v_i(g(\tilde{v}_i, v_{-i})) + \sum_{j\neq i} v_j(g(\tilde{v}_i, v_{-i})) - C(g(\tilde{v}_i, v_{-i})), \end{aligned} \tag{24}$$

where

$$g(\tilde{v}_i, v_{-i}) \in \underset{g}{\operatorname{argmax}} \left(\tilde{v}_i(g) + \textstyle\sum_{j\neq i} v_j(g) - C(g)\right) \tag{25}$$

- Reporting $\tilde{v}_i = v_i$ (truth-telling) is the best response!

VCG mechanism: feasibility

Designing the lump-sum tax

- Mechanism is only feasible if $\sum_{i=1}^{N} c_i(\tilde{v}) \geq C(g(\tilde{v}))$ for any $\tilde{v} := (\tilde{v}_i)_{i=1}^{N}$, which is equivalent to

$$\sum_{i=1}^{N} \bar{t}_i(\tilde{v}) \geq (N-1) \left(\sum_{i=1}^{N} \tilde{v}_i(g(\tilde{v})) - C(g(\tilde{v})) \right) \tag{26}$$

  or

$$\sum_{i=1}^{N} \left\{ \bar{t}_i(\tilde{v}) - \left(\tilde{v}_i(g(\tilde{v})) - \frac{N-1}{N} C(g(\tilde{v})) \right) \right\} \geq 0 \tag{27}$$

- Feasibility guaranteed by setting

$$\bar{t}_i(\tilde{v}) := \max_{g} \left(\sum_{j \neq i} \tilde{v}_j(g) - \frac{N-1}{N} C(g) \right) \tag{28}$$

VCG mechanism: inefficiency

Government's budget surplus

- VCG mechanism successfully encourages people to reveal their preference
- Government's budget, on the other hand, is not balanced in general
- Budget surplus: $\sum_{i=1}^{N} c_i(\tilde{v}) - C(g(\tilde{v})) \geq 0$

Cost of information

- Surplus cannot be transferred back to consumers because that would provide an incentive to tell a lie
- Surplus, if any, has to be wasted by design
- Pareto efficiency needs to be given up if we want to make the mechanism demand-revealing
- Can be interpreted as the cost of information