Intermediate public economics 5
Externalities

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Contents

Externality

Welfare theorem reconsidered

- Welfare theorem shows that efficiency will be (automagically) achieved at competitive equilibrium
- This is not necessarily the case in the presence of what we call externalities
- Externality is one primary reason for governmental intervention being justified

Definition

- We say that there is an externality if an action of one agent directly affects other agents in the economy
- By 'directly,' we mean 'not through a change of price'
- In other words, an externality is an interaction among agents that is external to the market

Real-world examples

Negative externalities

- Neighbor's consumption of loud music late at night
- Water pollution due to the discharges of an upstream factory
- Individual's abuse of antibiotics (which has the risk of making bacteria resistant to antibiotics)
- Keeping up with the Jones (positinal externality)

Positive externalities

- Maintaining a garden that is attractive to neighbors
- Pleasant smell of baking bread at a local bakery
- Becoming a member of social network sites or learning languages (network externality)
- Individual's investment in education

Pure exchange economy w/o externality

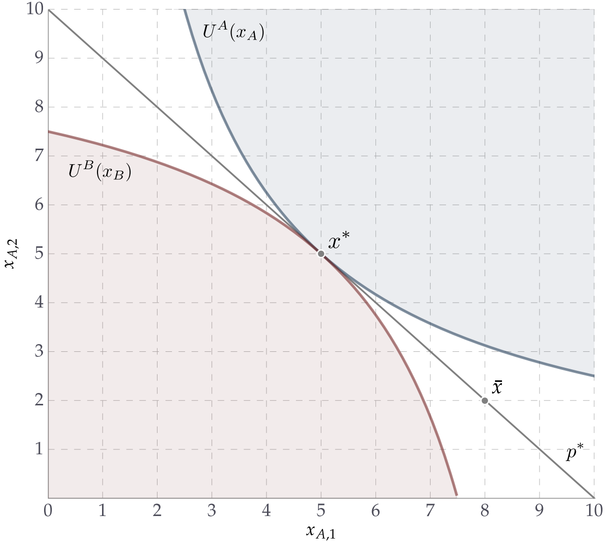
Setup

- Two people ($i \in \{A, B\}$) exchanging two goods
- Utility function: $U^i(x_i)$ where $x_i := (x_{i,1}, x_{i,2})$
- Initial endowment: $(\bar{x}_{i,1}, \bar{x}_{i,2})$

Competitive equilibrium

- $x^* = (x_A^*, x_B^*) \in \mathbb{R}_+^4$ is a competitive equilibrium if
  1. there exists $p^* \in \mathbb{R}_{++}$ such that for each $i \in \{A, B\}$,
  $$x_i^* \in \operatorname{argmax} U^i(x_i) \text{ s.t. } p^* x_{i,1} + x_{i,2} \leq p^* \bar{x}_{i,1} + \bar{x}_{i,2}, \quad (1)$$
  2. and x^* clears the markets, i.e.,
  $$\textstyle\sum_{i \in \{A,B\}} x_{i,l}^* = \sum_{i \in \{A,B\}} \bar{x}_{i,l} \quad \forall l \in \{1, 2\}. \quad (2)$$
- Welfare theorem suggests that x^* is Pareto efficient

Competitive equilibrium



Example 1

Quasi-linear utility function

- $U^i(x_i) := \ln(x_{i,1}) + x_{i,2}$ for both $i \in \{A, B\}$
- Utility-maximization condition implies

$$x_{i,1}^* = (p^*)^{-1} \text{ and } x_{i,2}^* = p^* \bar{x}_{i,1} + \bar{x}_{i,2} - 1 \tag{3}$$

- Market-clearing condition then implies

$$p^* = 2\bar{X}_1^{-1} \text{ where } \bar{X}_1 := \sum_i \bar{x}_{i,1} \tag{4}$$

- Therefore,

$$x_i^* = \left(\frac{1}{2}\bar{X}_1, \frac{\bar{x}_{i,1} - \bar{x}_{j,1}}{\bar{X}_1} + \bar{x}_{i,2} \right) \tag{5}$$

- Observe that the indifference curves touch to each other at the equilibrium level of consumption

Introducing externality

Setup

- A's consumption of good 1 causes an external effect $E(x_{A,1})$ with $E'(x_{A,1}) > 0$ (loud music late at night)
- B's (true) utility V^B is negatively affected by E

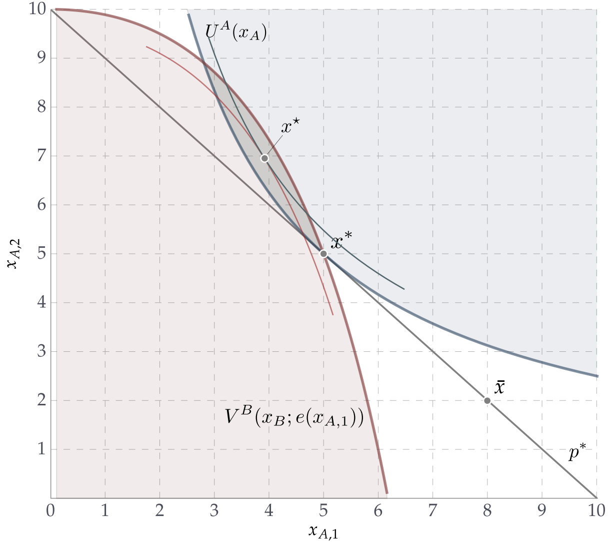
$$V^B(x_B; E) := U^B(x_B) - \phi(E) \tag{6}$$

for some strictly increasing function ϕ

Market failure

- Here E is an externality (i.e., it directly affects B)
- B hates A's consumption of good 1 but she has no way of conveying that information through market
- This is why market fails in the presence of externality

Equilibrium with externality



Guided exercise

Proving the inefficiency

- For each $x = (x_A, x_B)$, define $\Delta(x) \in \mathbb{R}$ by

$$\Delta(x) := \frac{U_1^B(x_B) + \phi'(E(x_{A,1}))E'(x_{A,1})}{U_2^B(x_B)} - \frac{U_1^A(x_A)}{U_2^A(x_A)} \qquad (7)$$

- $\Delta(x)$ is NV of transferring good 1 from A to B
- Notice that $\Delta(x^*) > 0$ at eqm $x^* := (x_A^*, x_B^*)$
- Consider the following reallocation:

$$x'_A := x_A^* + (-\varepsilon, \delta(\varepsilon)) \text{ and } x'_B := x_B^* + (\varepsilon, -\delta(\varepsilon)), \qquad (8)$$

where

$$\delta(\varepsilon) := (U_1^A(x_A^*)/U_2^A(x_A^*) + \Delta(x^*)/2)\varepsilon \qquad (9)$$

- Then $x' := (x'_A, x'_B)$ is feasible and Pareto dominates x^* for sufficiently small $\varepsilon > 0$

Pareto efficient allocations

Necessary condition

- In general, Pareto improvement is possible if $\Delta(x) \neq 0$
- An allocation $x^\star$ is Pareto efficient only if $\Delta(x^\star) = 0$, or

$$\frac{U_1^B(x_B^\star) + \phi'(E(x_{A,1}^\star))E'(x_{A,1}^\star)}{U_2^B(x_B^\star)} = \frac{U_1^A(x_A^\star)}{U_2^A(x_A^\star)} \tag{10}$$

- Competitive equilibrium would never be Pareto efficient unless $E' = 0$ (which is the case of no externality)

Geometric interpretation

- $\Delta(x)$ is the difference between marginal rates of substitution of A and B
- Hence, $\Delta(x^\star) = 0$ requires that indifference curves in the Edgeworth box must touch to each other at $x^\star$

Alternative interpretation

Disparity between social and private cost

- Social benefit (in units of good 2) of increasing $x_{A,1}$:

$$MSB(x) := \frac{U_1^A(x_A)}{U_2^A(x_A)} \tag{11}$$

- Social cost of increasing $x_{A,1}$ (and decreasing $x_{B,1}$):

$$MSC(x) := \frac{U_1^B(x_B) + \phi'(E(x_{A,1}))E'(x_{A,1})}{U_2^B(x_B)} \tag{12}$$

- $x^\star$ is Pareto efficient only if $MSB(x^\star) = MSC(x^\star)$
- At eqm, however,

$$MSB(x^*) = p^* < MSC(x^*), \tag{13}$$

where p^* is the private cost (for A) of increasing $x_{A,1}$

Example 1 (with externality)

Quasi-linear utility function

- $U^i(x_i) := \ln(x_{i,1}) + x_{i,2}$ for both $i \in \{A, B\}$
- Simply assume $E(x_{A,1}) := x_{A,1}$
- Also put $\phi(E) := \alpha \ln(E)$ for some $\alpha \in (0, 1)$

Inefficiency of the competitive equilibrium

- Equilibrium is characterized as before, in particular,

$$x^*_{A,1} = (1/2)\bar{X}_1 \tag{14}$$

- Indifference curves cross each other ($\Delta(x^*) \neq 0$)
- If $x^\star$ is Pareto efficient, it must satisfy $\Delta(x^\star) = 0$, or

$$x^\star_{A,1} = \frac{1-\alpha}{2-\alpha}\bar{X}_1 < \frac{1}{2}\bar{X}_1 = x^*_{A,1}, \tag{15}$$

meaning that good 1 is overconsumed by A at eqm

Production economy w/o externality

Setup

- Firm $j \in \{1,2\}$ produces good j using labor ($x_j = f_j(l_j)$)
- Single consumer with utility $U(x_1, x_2)$ and endowment $\bar{l}$

Competitive equilibrium

- $x^* = (x_1^*, x_2^*) \in \mathbb{R}_+^2$ is a competitive equilibrium if
  1. there exists $(p^*, w^*) \in \mathbb{R}_{++}^2$ such that

$$x^* \in \operatorname{argmax} U(x) \text{ s.t. } p^* x_1 + x_2 \leq w^* \bar{l} + \textstyle\sum_j \pi_j^*,$$

$$l_j^* \in \operatorname{argmax} \pi_j = \begin{cases} p^* f_1(l_1) - w^* l_1 & \text{for } j = 1 \\ f_2(l_2) - w^* l_2 & \text{for } j = 2, \end{cases} \tag{16}$$

  2. and x^* clears the markets, i.e.,

$$\textstyle\sum_j l_j^* = \bar{l} \text{ and } x_j^* = f_j(l_j^*) \quad \forall j \in \{1,2\} \tag{17}$$

Efficiency of competitive equilibrium

Production possibility set

- Define the production possibility set $X \subseteq \mathbb{R}^2_+$ by

$$X := \{x \in \mathbb{R}^2_+ \mid x_j \le f_j(l_j) \text{ and } \textstyle\sum_j l_j \le \bar{l}\} \tag{18}$$

- Set of all technically feasible production plans

Efficiency

- At eqm,

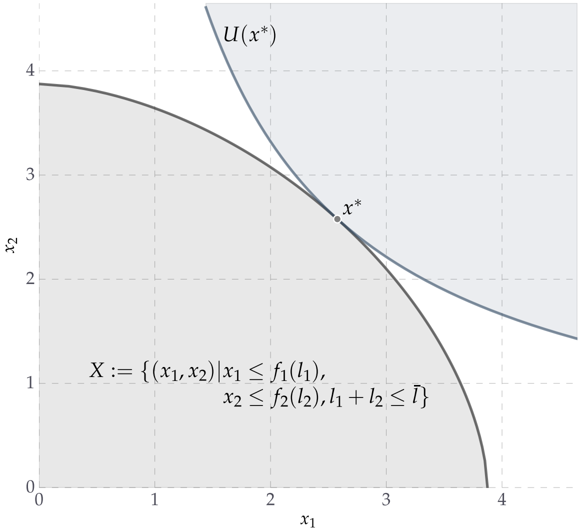
$$MRS(x^*) := \frac{U_1(x^*)}{U_2(x^*)} = p^* = \frac{f_2'(l_2^*)}{f_1'(l_1^*)} =: MRT(x^*) \tag{19}$$

and

$$\textstyle\sum_j l_j^* = \bar{l} \text{ and } x_j^* = f_j(l_j^*) \quad \forall j \in \{1,2\} \tag{20}$$

- (20) means that x^* is on the edge (frontier) of X
- (19) implies that indifference curve touches to X at x^*

Equilibrium in production economy



Example 2

Linear technology & quasi-linear utility

- $f_j(l_j) := a_j l_j$ for some $a_j \in \mathbb{R}_{++}$ for each $j \in \{1, 2\}$
- Specify $U(x_1, x2) := \ln(x_1) + x_2$
- Assume $\bar{l} > 1/a_2$

Solving for the equilibrium

- It follows from the profit maximization behavior that

$$w^* = a_2 \text{ and } p^* = a_2/a_1 \tag{21}$$

- Utility maximization then implies

$$x_1^* = 1/p^* = a_1/a_2 \tag{22}$$

- Use the market-clearing condition to obtain

$$x_2^* = a_2\bar{l} - 1 \tag{23}$$

Introducing production externality

Setup

- Production of good 2 (say, education) causes an external effect
- This external effect bumps up the productivity of firm 1

$$x_1 = \tilde{f}_1(l_1; x_2) := \phi(x_2) f_1(l_1) \tag{24}$$

for some strictly increasing function ϕ with $\phi(0) = 1$

- Firm 1 benefits from the production of good 2 but that information is not reflected in the market price

Marginal rate of transformation

- MRT (slope of PPF) is now given by

$$MRT(x) := \frac{f_2'(l_2)}{\tilde{f}_1'(l_1; x_2) - \phi'(x_2) f_2'(l_2) f_1(l_1)} \tag{25}$$

Guided exercise

Proving the inefficiency

- At eqm x^*,

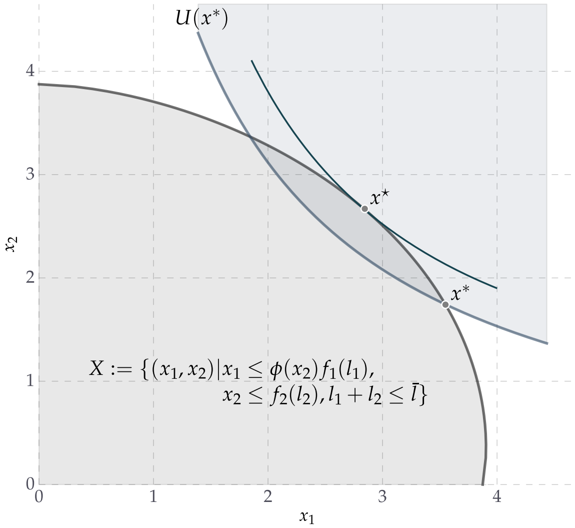
$$MRS(x^*) = \frac{U_1(x^*)}{U_2(x^*)} = p^* = \frac{f_2'(l_2^*)}{\tilde{f}_1'(l_1^*, x_2^*)} \tag{26}$$
$$< \frac{f_2'(l_2^*)}{\tilde{f}_1'(l_1^*; x_2^*) - \phi'(x_2^*) f_2'(l_2^*) f_1(l_1^*)} = MRT(x^*)$$

- This indicates that reallocating resource (labor) from firm 1 to firm 2 will achieve Pareto improvement
- Consider $x_1' := \tilde{f}_1(l_1', x_2')$, $x_2' := f_2(l_2')$ where

$$l_1' := l_1^* - \varepsilon \text{ and } l_2' := l_2^* + \varepsilon \tag{27}$$

- Then $x' := (x_1', x_2')$ is feasible and Pareto dominates x^* for sufficiently small $\varepsilon > 0$

Production externality



Pareto efficient allocations

Necessary (and sufficient) condition

- Pareto improvement is possible if $MRS(x) \neq MRT(x)$
- An allocation $x^\star$ is Pareto efficient (if and) only if

$$\frac{U_1(x^\star)}{U_2(x^\star)} = \frac{f_2'(l_2^\star)}{\tilde{f}_1'(l_1^\star; x_2^\star) - \phi'(x_2^\star) f_2'(l_2^\star) f_1(l_1^\star)} \tag{28}$$

- Pareto efficient allocation is (under the standard assumption) unique in this economy because there is only one consumer

Geometric interpretation

- $MRS(x) \neq MRT(x)$ means indifference curve and production possibility frontier (PPF) cross at x
- $MRS(x^\star) = MRT(x^\star)$ requires that indifference curve and PPF must touch to each other at $x^\star$

Alternative interpretation

Disparity between social and private benefit

- Social benefit (in units of good 2) of increasing l_2:

$$MSB(x) := f_2'(l_2) + \frac{U_1(x)}{U_2(x)}\phi'(x_2)f_1(l_1)f_2'(l_2) \tag{29}$$

- Social cost of increasing l_2 (and decreasing l_1):

$$MSC(x) := \frac{U_1(x)}{U_2(x)}\tilde{f}_1'(l_1; x_2) \tag{30}$$

- At eqm,

$$\begin{aligned} MSB(x^*) > f_2'(l_2^*) = w^* = p^*\tilde{f}_1'(l_1^*; x_2^*) \\ = \frac{U_1(x^*)}{U_2(x^*)}\tilde{f}_1'(l_1^*; x_2^*) = MSC(x^*), \end{aligned} \tag{31}$$

where $f_2'(l_2^*)$ is firm 2' private benefit of increasing l_2

Example 2 (with externality)

Linear technology & quasi-linear utility

- Assume linear technology and quasi-linear utility
- Specify $\phi(x_2) := e^{x_2}$ (i.e., exponential function)

Inefficiency of the competitive equilibrium

- Equilibrium is characterized by

$$x_1^* = \frac{a_1}{a_2} e^{a_2 \bar{l} - 1} \text{ and } x_2^* = a_2 \bar{l} - 1 \tag{32}$$

- Observe $MRS(x^*) < \infty = MRT(x^*)$
- If $x^\star$ is Pareto efficient, it must satisfy

$$MRS(x^\star) = MRT(x^\star) \implies x_2^\star = a_2 \bar{l} - \frac{1}{2} > x_2^*, \tag{33}$$

meaning that good 2 is underproduced at eqm

Internalization

Removing the inefficiency

- Externality is a source of inefficiency
- We say that an externality is internalized when the associated inefficiency is removed
- Removing inefficiency often requires governmental intervention

Options for internalization

- Command and control (i.e., standard setting) is an obvious option, but is not of interest here
- We consider the following three options:
  1. price regulation
  2. quantity regulation
  3. market creation (or bargaining)

Tax and subsidy

The idea

- Primary reason for externality-induced inefficiency is the disparity between private and social costs
- Agents take into account the private cost of their actions (through market price), but ignores the social cost (which is not reflected in the market price)
- Just let them know this fact by adding the ignored part of the social cost to the market price

Some remarks

- Tax revenue should be brought back to consumers in some non-distortionary way
- For positive externalities, use subsidies
- Budget for the subsidy should be financed in some non-distortionary way

Pure exchange economy (with tax)

Taxation on the external effect

- Denote by τ a per-unit tax on the external effect $E(x_{A,1})$ (in units of good 2)
- Tax revenue will then be $\tau E(x_{A,1})$
- Let $T_i \in \mathbb{R}$ be a lump-sum transfer to $i \in \{A, B\}$ from government, which at equilibrium must satisfy

$$T_A + T_B = \tau E(x_{A,1}) \tag{34}$$

Government's problem

- Policy instruments are τ, T_A, and T_B
- Government can set the values of these variables as long as (34) is satisfied
- Degree of freedom is therefore 2 (say, τ and T_A)
- Equilibrium is then a function of (τ, T_A)

Competitive equilibrium (with tax)

Characterizing equilibrium

- First-order conditions:

$$\frac{U_1^A(x_A^*)}{U_2^A(x_A^*)} = p^* + \tau \text{ and } p^* = \frac{U_1^B(x_B^*)}{U_2^B(x_B^*)} \qquad (35)$$

- Consumers' budget constraints:

$$p^* x_{A,1}^* + x_{A,2}^* = p^* \bar{x}_{A,1} + \bar{x}_{A,2} - \tau E(x_{A,1}^*) + T_A \qquad (36)$$

$$p^* x_{B,1}^* + x_{B,2}^* = p^* \bar{x}_{B,1} + \bar{x}_{B,2} + T_B \qquad (37)$$

- Government's budget constraint:

$$T_A + T_B = \tau E(x_{A,1}^*) \qquad (38)$$

- Market-clearing condition:

$$\textstyle\sum_{i \in \{A,B\}} x_{i,l}^* = \sum_{i \in \{A,B\}} \bar{x}_{i,l} \quad \forall l \in \{1,2\} \qquad (39)$$

Designing a tax scheme

Pigouvian tax

- Let $x^\star$ be a Pareto efficient allocation (our 'target')
- Set the tax rate $\tau^\star$ as

$$\tau^\star := \frac{\phi'(E(x^\star_{A,1}))E'(x^\star_{A,1})}{U_2^B(x_B^\star)} \tag{40}$$

- Set the transfer $T_A^\star$ as

$$T_A^\star := \frac{U_1^B(x_B^\star)}{U_2^B(x_B^\star)} x^\star_{A,1} + x^\star_{A,2} - \frac{U_1^B(x_B^\star)}{U_2^B(x_B^\star)} \bar{x}_{A,1} - \bar{x}_{A,2} + \tau^\star E(x^\star_{A,1}) \tag{41}$$

- Then the eqm x^* under the scheme $(\tau^\star, T_A^\star)$ coincides with the target allocation $x^\star$! (b/c (10) is satisfied)
- This tax-transfer scheme is called the Pigouvian tax

Remarks on Pigouvian tax

How does it work?

- Reverse engineering, in essence
- Any Pareto efficient allocation can be supported as a competitive equilibrium under an appropriately designed Pigouvian tax-transfer scheme
- Just like the second welfare theorem

Difficulties

- Theoretically beautiful, but not easy to implement (again, as is the second welfare theorem)
- Information about preference (U^i and ϕ) is required
- In general, Pigouvian tax rate needs to be differentiated across agents (depending on how much your neighbor dislikes the external effect you generate)

Example 1 (with Pigouvian tax)

Quasi-linear utility

- Recall Example 1 with consumption externality
- Observe that the following allocation is Pareto efficient:

$$(x^\star_{A,1}, x^\star_{A,2}) := \left(\frac{1-\alpha}{2-\alpha}\bar{X}_1, \frac{(1-\alpha)\bar{x}_{A,1} - \bar{x}_{B,1}}{\bar{X}_1} + \bar{x}_{A,2}\right) \quad (42)$$

and $(x^\star_{B,1}, x^\star_{B,2}) := (\bar{X}_1 - x^\star_{A,1}, \bar{X}_2 - x^\star_{A,2})$

Computing Pigouvian tax rate

- This allocation can be supported as an equilibrium if we set

$$\tau^\star := \frac{\alpha}{x^\star_{A,1}} = \frac{\alpha(2-\alpha)}{(1-\alpha)\bar{X}_1} \quad (43)$$

and

$$T^\star_A := 0 \text{ and } T^\star_B := \alpha \quad (44)$$

Production economy (with subsidy)

Subsidy for good 2

- Let τ be a per-unit subsidy on sales of good 2
- Firm 2's profit maximization problem is then

$$\max \pi_2 := (1+\tau)x_2 - wl_2 \text{ where } x_2 = f_2(l_2) \qquad (45)$$

- Total amount of subsidy paid by government is τx_2, which should be financed through lump-sum taxation T on consumer

Government's problem

- Policy instruments are τ and T
- Government's budget constraint $\tau x_2 = T$ must be satisfied (degree of freedom is hence 1, say τ)
- Equilibrium is then a function of τ

Competitive equilibrium (with subsidy)

Characterizing equilibrium

- Consumer's first-order condition:

$$U_1(x^*)/U_2^A(x^*) = p^* \tag{46}$$

- Consumer' budget constraint:

$$p^* x_1^* + x_2^* = w^* \bar{l} + \sum_j \pi_j^* - T \tag{47}$$

- Firms' first-order conditions:

$$p^* \phi(x_2^*) f_1'(l_1^*) - w^* = 0 \text{ and } (1+\tau) f_2'(l_2^*) - w^* = 0 \tag{48}$$

- Market-clearing condition: $l_1^* + l_2^* = \bar{l}$ and

$$x_1^* = \phi(x_2^*) f_1(l_1^*) \text{ and } x_2^* = f_2(l_2^*) \tag{49}$$

- Government's budget constraint: $\tau x_2^* = T$

Designing a subsidy scheme

Pigouvian subsidy

- Let $x^\star$ be the Pareto efficient allocation (our 'target')
- Set the subsidy rate $\tau^\star$ as

$$\tau^\star := \frac{U_1(x^\star)}{U_2(x^\star)} \frac{\phi'(x_2^\star)}{\phi(x_2^\star)} x_1^\star \tag{50}$$

- Set $T^\star := \tau^\star x_2^\star$
- Then the eqm x^* under this subsidy scheme coincides with the target allocation $x^\star$! (because (28) is satisfied)

Alternative way

- You could instead subsidize production factor (labor) for good 2 to facilitate the production of the otherwise underproduced good

Example 2 (with Pigouvian subsidy)

Linear technology & quasi-linear utility

- We already know the following allocation is Pareto efficient:

$$(x_1^\star, x_2^\star) := \left(\frac{1}{2} \frac{a_1}{a_2} e^{a_2 \bar{l} - \frac{1}{2}}, a_2 \bar{l} - \frac{1}{2} \right) \tag{51}$$

Computing Pigouvian subsidy rate

- It should be easy to see that setting

$$\tau^\star := 1 \tag{52}$$

  will do the trick

- Setting the correct subsidy rate requires the information about technology as well as preference, both of which are often private information (unknown to government)

Tax on externality: in practice

Aiming at Pareto improvement

- Setting the correct Pigouvian tax/subsidy rate is difficult (if not impossible) in terms of information required
- But introducing some tax system for internalizing externalities is still useful
- Such a tax/subsidy, if appropriately designed, is likely to achieve Pareto improvement (even though Pareto efficiency is not attained)

Adjustment over time

- Government can adjust the tax/subsidy rate over time
- Start a relatively low rate and then change it depending on how people/firms react to the original rate
- Hopefully, the adjustment process converges at some point

Cost-minimization effect

Cost of reducing/increasing external effects

- When there are multiple sources of an externality, the cost of reducing/increasing the negative/positive external effect is often different across different sources
- Reducing one unit of pollutant might be very difficult for one firm, but could be quite easy for another
- This information is typically private (i.e., not public)

Positive rate of tax/subsidy minimize the total cost

- Obviously not efficient if the same amount of externality-adjustment is required for all sources
- Tax/subsidy, once introduced, equalizes the marginal costs of adjusting the external effect among different sources
- No private information required

Illustration of cost-minimization effect

Two polluting firms

- Firm $j \in \{1, 2\}$ produces good j using labor ($x_j = f_j(l_j)$)
- Pollution $\phi(x_j)$ produced as a byproduct
- Pollution abatement a_j is possible, but requires extra labor $\tilde{l}_j = c_j(a_j)$ with $c_j(0) = 0$, $c_j' > 0$, and $c_j'' \leq 0$
- Net pollution from firm j is $z_j = \phi(x_j) - a_j$

Firms profit maximization

- Denote by τ a tax on the pollution
- Then the firm j's problem is

$$\max \pi_j := p_j x_j - w(l_j + \tilde{l}_j) - \tau z_j \tag{53}$$
$$\text{s.t. } x_j = f_j(l_j),\ z_j = \phi(x_j) - a_j, \text{ and } \tilde{l}_j = c_j(a_j)$$

Illustration of cost-minimization effect

Marginal cost equalized

- Profit-maximization directly implies

$$c_1'(a_1^*) = \frac{\tau}{w} = c_2'(a_2^*), \tag{54}$$

  meaning that the marginal abatement costs (in units of labor) are equalized across firms
- This implies that the cost of reducing $A^* := \sum_j a_j^*$ unit of pollutant is minimized at the social level

You don't see why?

- If (54) is not satisfied, reallocating labor from one firm to another will achieve the same amount of pollution reduction at a strictly lower cost
- Assume $c_1'(a_1) < c_2'(a_2)$ and work it out yourself

Quantity regulation

Regulating quantity

- Another way of internalizing externalities is to regulate quantity (so called 'cap-and-trade' policy)
- Equivalent to creating a market where the quantity of externality-causing goods can be traded among stakeholders
- A fixed amount of permits issued by the regulator, allocated to stakeholders, and then traded
- Price is determined in the market

Real-world examples

- Emission trading program for sulfur dioxide in US, initiated by the Clean Air Act of 1990
- EU emission trading scheme for carbon dioxide (2005–)

Pure exchange economy (with cap)

Cap and allocation

- Government issues a fixed amount $\bar{E}$ of permits (the right to enjoy laud music for $\bar{E}$ minutes late at night)
- Allocate $\theta\bar{E}$ to A ('polluter') and $(1-\theta)\bar{E}$ to B ('victim') for some $\theta \in [0, 1]$
- Policy instruments for government are $\bar{E}$ and θ

Trade

- Permits are traded with p_e being its price
- Denote by E_i the amount of permits possessed by $i \in \{A, B\}$ so that

$$E_A + E_B = \bar{E} \tag{55}$$

- Consumer A buys (sells) $E_A - \theta\bar{E}$ while consumer B sells (buys) $(1-\theta)\bar{E} - E_B$

Pure exchange economy (with cap)

Consumer A's problem

- Consumer A chooses $(x_{A,1}, x_{A,2}, E_A)$ so as to maximize $U^A(x_A)$ subject to

$$px_{A,1} + x_{A,2} + p_e E_A = p\bar{x}_{A,1} + \bar{x}_{A,2} + p_e \theta \bar{E} \tag{56}$$

and

$$E(x_{A,1}) = E_A \tag{57}$$

Consumer B's problem

- Similarly, consumer B chooses $(x_{B,1}, x_{B,2}, E_B)$ so as to maximize $V^B(x_B; E_A)$ subject to

$$px_{B,1} + x_{B,2} + p_e E_B = p\bar{x}_{B,1} + \bar{x}_{B,2} + p_e(1-\theta)\bar{E} \tag{58}$$

- Permit E_B (if positive) will never be used because B does not cause externality

Competitive equilibrium (with cap)

Characterizing equilibrium

- First-order conditions:

$$\frac{U_1^A(x_A^*)}{U_2^A(x_A^*)} = p^* + p_e^* E'(x_{A,1}^*) \text{ and } p^* = \frac{U_1^B(x_B^*)}{U_2^B(x_B^*)} \tag{59}$$

- Demand for permits: $E_A^* = E(x_{A,1}^*)$ and $E_B^* = 0$
- Consumers' budget constraints:

$$p^* x_{A,1}^* + x_{A,2}^* + p_e^* E_A^* = p^* \bar{x}_{A,1} + \bar{x}_{A,2} + p_e^* \theta \bar{E} \tag{60}$$

$$p^* x_{B,1}^* + x_{B,2}^* + p_e^* E_B^* = p^* \bar{x}_{B,1} + \bar{x}_{B,2} + p_e^* (1-\theta) \bar{E} \tag{61}$$

- Market-clearing conditions:

$$\sum_i x_{i,l}^* = \sum_i \bar{x}_{i,l} \; \forall l \in \{1,2\} \quad \text{and} \quad \sum_i E_i^* = \bar{E} \tag{62}$$

Designing a cap-and-trade scheme

Government's problem

- Design a policy $(\bar{E}, \theta)$ to achieve Pareto efficiency
- Let $x^\star$ be a Pareto efficient allocation (our 'target')
- Set $\bar{E}^\star$ and $\theta^\star$ as $\bar{E}^\star := E(x^\star_{A,1})$ and

$$\theta^\star := 1 - \frac{\frac{U_1^B(x_B^\star)}{U_2^B(x_B^\star)}(x^\star_{B,1} - \bar{x}_{B,1}) + x^\star_{B,2} - \bar{x}_{B,2}}{\left(\frac{U_1^A(x_A^\star)}{U_2^A(x_A^\star)} - \frac{U_1^B(x_B^\star)}{U_2^B(x_B^\star)}\right)\frac{\bar{E}^\star}{E'(x^\star_{A,1})}} \tag{63}$$

- Then the eqm x^* coincides with the target allocation $x^\star$!

But wait ...

- We need to know what we cannot know (preference)
- Equivalent to tax-transfer scheme in terms of information required

Example 1 (with efficient cap)

Quasi-linear utility

- Recall Example 1, where a Pareto efficient allocation is

$$(x^\star_{A,1}, x^\star_{A,2}) := \left(\frac{1-\alpha}{2-\alpha} \bar{X}_1, \frac{(1-\alpha)\bar{x}_{A,1} - \bar{x}_{B,1}}{\bar{X}_1} + \bar{x}_{A,2} \right) \quad (64)$$

and $(x^\star_{B,1}, x^\star_{B,2}) := (\bar{X}_1 - x^\star_{A,1}, \bar{X}_2 - x^\star_{A,2})$

Computing efficient cap and permit allocation

- This allocation can be supported as an equilibrium if we set

$$\bar{E}^\star := x^\star_{A,1} = \frac{1-\alpha}{2-\alpha} \bar{X}_1 \quad (65)$$

and

$$\theta^\star := 0 \quad (66)$$

- Policy $\theta^\star = 0$ in effect transfers income from A to B

Cap-and-trade policy: in practice

Aiming at Pareto improvement

- Setting the correct amount of total permits is difficult (if not impossible) in terms of information required
- But introducing some cap on externality-causing goods is still useful because such a policy is likely to achieve Pareto improvement

Cost-minimization effect

- When there are multiple sources of an externality, the cost of reducing/increasing the negative/positive external effect is often different across different sources
- Cap-and-trade scheme, once introduced, equalizes the marginal costs of adjusting the external effect among different sources
- Hence, cost minimization follows

Illustration of cost-minimization effect

Two polluting firms

- Firm $j \in \{1, 2\}$ produces good j using labor ($x_j = f_j(l_j)$)
- Pollution $\phi(x_j)$ produced as a byproduct
- Pollution abatement a_j is possible, but requires extra labor $\tilde{l}_j = c_j(a_j)$ with $c_j(0) = 0$, $c_j' > 0$, and $c_j'' \leq 0$
- Net pollution from firm j is $z_j = \phi(x_j) - a_j$

Firms profit maximization

- Denote by $\bar{z}$ the total amount of permits issued and $\theta_j \in [0, 1]$ be such that $\theta_1 + \theta_2 = 1$
- Then the firm j's problem is

$$\max \pi_j := p_j x_j - w(l_j + \tilde{l}_j) - p_z(z_j - \theta_j \bar{z}) \tag{67}$$
$$\text{s.t.} \quad x_j = f_j(l_j),\ z_j = \phi(x_j) - a_j, \text{ and } \tilde{l}_j = c_j(a_j)$$

Illustration of cost-minimization effect

Marginal cost equalized

- Profit-maximization directly implies

$$c_1'(a_1^*) = \frac{p_z}{w} = c_2'(a_2^*), \tag{68}$$

  meaning that the marginal abatement costs (in units of labor) are equalized across firms
- This implies that the cost of reducing $A^* := \sum_j a_j^*$ unit of pollutant is minimized at the social level

You don't see why?

- If (68) is not satisfied, reallocating labor from one firm to another will achieve the same amount of pollution reduction at a strictly lower cost
- Assume $c_1'(a_1) < c_2'(a_2)$ and work it out yourself

Coase theorem: the idea

Difficulty in designing policies

- Clearly, the problem is that we do not know how to choose the total amount of permits
- Information required for designing optimal policies is often private, unknown to policy makers
- But do we really need to know that private information?

Just let 'them' decide

- On second thought, the total amount of permits does not have to be determined by policy makers
- Just let stakeholders decide how much permits should be issued in the market because they have all the information required for achieving efficiency
- This is the central idea lying behind the so-called Coase Theorem

Coase theorem: illustration

Let the 'victim' decide

- Recall our pure-exchange-economy setup
- A chooses $(x_{A,1}, x_{A,2}, E_A)$ to maximize $U^A(x_A)$ s.t.

$$px_{A,1} + x_{A,2} + p_e E_A = p\bar{x}_{A,1} + \bar{x}_{A,2} \quad (69)$$

  and $E(x_{A,1}) = E_A$
- B chooses $(x_{B,1}, x_{B,2}, \bar{E})$ to maximize $V^B(x_B; \bar{E})$ s.t.

$$px_{B,1} + x_{B,2} = p\bar{x}_{B,1} + \bar{x}_{B,2} + p_e \bar{E} \quad (70)$$

- At eqm, market should be cleared in the sense that

$$E_A = \bar{E} \quad (71)$$

- Notice that government does not have to choose $\bar{E}$
- B decides $\bar{E}$, taking into account how it affects her utility

Coase theorem: illustration (cont'd)

Characterizing equilibrium

- First-order conditions:

$$\frac{U_1^A(x_A^*)}{U_2^A(x_A^*)} = p^* + p_e^* E'(x_{A,1}^*) \text{ and } p^* = \frac{U_1^B(x_B^*)}{U_2^B(x_B^*)} \tag{72}$$

$$\frac{V_E^B(x_B^*; \bar{E}^*)}{V_2^B(x_B^*; \bar{E}^*)} = -\frac{\phi'(\bar{E}^*)}{U_2^B(x_B^*)} = -p_e^* \tag{73}$$

- Consumers' budget constraints:

$$p^* x_{A,1}^* + x_{A,2}^* + p_e^* E(x_{A,1}^*) = p^* \bar{x}_{A,1} + \bar{x}_{A,2} \tag{74}$$

$$p^* x_{B,1}^* + x_{B,2}^* = p^* \bar{x}_{B,1} + \bar{x}_{B,2} + p_e^* \bar{E}^* \tag{75}$$

- Market-clearing conditions:

$$\sum_i x_{i,l}^* = \sum_i \bar{x}_{i,l} \, \forall l \in \{1, 2\} \quad \text{and} \quad E(x_{A,1}^*) = \bar{E}^* \tag{76}$$

Coase theorem: illustration (cont'd)

Efficiency restored

- Combining these conditions yields

$$\frac{U_1^A(x_A^*)}{U_2^A(x_A^*)} = p^* + p_e^* E'(x_{A,1}^*) = \frac{U_1^B(x_B^*) + \phi'(E(x_{A,1}^*))E'(x_{A,1}^*)}{U_2^B(x_B^*)}, \tag{77}$$

  which is nothing but the efficiency condition (10)!

Not surprising, right?

- Demand of permit captures A's private information
- Supply of permit captures B's private information
- As a result, equilibrium price p_e^* correctly reflects how much B dislikes the loud music (benefit of reducing E) as well as how much A likes it (cost of reducing E)

Coase theorem: formal statement

Coase theorem

- Consider a competitive economy with complete information and zero transaction costs
- If property rights are all well defined in the economy,
  1. the equilibrium allocation will be Pareto efficient, and
  2. this result does not depend on how the property rights are defined and allocated

Property rights

- Government might want to entitle people to the right to enjoy silence late at night
- Could be defined in a different way: the right to enjoy loud music late at night
- Property rights, no matter how they are defined, should clearly state who has what

Coase theorem: second part

Let the 'polluter' decide

- Right to enjoy loud music $E_0 \in \mathbb{R}_+$ entitled to A
- A chooses $(x_{A,1}, x_{A,2}, \bar{E})$ to maximize $U^A(x_A)$ s.t.

$$px_{A,1} + x_{A,2} = p\bar{x}_{A,1} + \bar{x}_{A,2} + p_e(E_0 - \bar{E}) \tag{78}$$

  and $E(x_{A,1}) = \bar{E}$
- B chooses $(x_{B,1}, x_{B,2}, E_B)$ to maximize $V^B(x_B; E_0 - E_B)$ s.t.

$$px_{B,1} + x_{B,2} + p_e E_B = p\bar{x}_{B,1} + \bar{x}_{B,2} \tag{79}$$

- At eqm, market should be cleared in the sense that

$$E_0 - \bar{E} = E_B \tag{80}$$

- Notice that E_0 can be chosen arbitrarily by government
- A decides $\bar{E}$, taking into account how it affects her utility

Coase theorem: second part (cont'd)

Characterizing equilibrium

- First-order conditions:

$$\frac{U_1^A(x_A^*)}{U_2^A(x_A^*)} = p^* + p_e^* E'(x_{A,1}^*) \text{ and } p^* = \frac{U_1^B(x_B^*)}{U_2^B(x_B^*)} \tag{81}$$

$$-\frac{V_E^B(x_B^*; E_0 - E_B^*)}{V_2^B(x_B^*; E_0 - E_B^*)} = \frac{\phi'(E_0 - E_B^*)}{U_2^B(x_B^*)} = p_e^* \tag{82}$$

- Consumers' budget constraints:

$$p^* x_{A,1}^* + x_{A,2}^* = p^* \bar{x}_{A,1} + \bar{x}_{A,2} + p_e^*(E_0 - E(x_{A,1}^*)) \tag{83}$$

$$p^* x_{B,1}^* + x_{B,2}^* + p_e^* E_B^* = p^* \bar{x}_{B,1} + \bar{x}_{B,2} \tag{84}$$

- Market-clearing conditions:

$$\sum_i x_{i,l}^* = \sum_i \bar{x}_{i,l} \, \forall l \in \{1, 2\} \quad \text{and} \quad E_0 - E(x_{A,1}^*) = E_B^*$$

Coase theorem: illustration (cont'd)

Efficiency restored

- Combining these conditions yields

$$\frac{U_1^A(x_A^*)}{U_2^A(x_A^*)} = p^* + p_e^* E'(x_{A,1}^*)$$
$$= \frac{U_1^B(x_B^*) + \phi'(E(x_{A,1}^*))E'(x_{A,1}^*)}{U_2^B(x_B^*)}, \tag{85}$$

which is equivalent to the efficiency condition (10)!

Same old story?

- Again, eqm price p_e^* correctly reflects how much B dislikes the loud music and how much A likes it
- This time, however, supply of permit (i.e., $E_0 - \bar{E}$) captures A's private information
- Demand of permit captures B's private information

Exercise

Setup

- Recall Example 1 with externality (loud music):
  - $U^i(x_i) := \ln(x_{i,1}) + x_{i,2}$ for both $i \in \{A, B\}$
  - $V^B(x_B; E) := U^B(x_B) - \phi(E)$
  - $E(x_{A,1}) := x_{A,1}$ and $\phi(E) := \alpha \ln(E)$ with $\alpha \in (0, 1)$

Question

- Consider first the case where the right to enjoy silence late at night is entitled to everybody
- Compute the competitive equilibrium when 'loud-music' permits are traded
- Is the equilibrium Pareto efficient?
- What if the right to enjoy loud music late at night (for E_0 hours) is entitled to everybody instead?

Coase theorem: implications

Any role of government?

- Coase theorem suggests that efficient outcomes may be achieved without active intervention of government
- All they need to do is to define property rights (the distributional consequence depends on how they are defined and allocated, though)
- No tax/subsicy nor cap-and-trade program required

Practical relevance

- Not easy to define property rights in a universally acceptable way (polluter pay or beneficiary pay)
- Transaction cost is high (even infinite in some cases), which is the very reason why the market for externality-causing goods does not exist!
- Often involves bargaining and hence strategic incentive