Intermediate public economics 4
Cost-benefit analysis

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Policy evaluation

Governmental policies

- Redistribution of wealth, taxation on goods, regulation against socially undesirable activities
- How would you evaluate these governmental policies?
- Governmental intervention will alter the allocation of resource, usually involving both cost and benefit

Cost and benefit

- Production of one good can be increased in return for decreasing another good
- Consumption level of one person can be increased at the cost of another person's consumption
- Comparing cost and benefit often requires inter-goods and inter-personal comparison

→ Need to compute the value of cost and benefit

Value of goods

What is the 'value'?

- Value of a good is not inherent in the good itself
- Only meaningful when the context is fixed
- A glass of drinkable water might have little value for you, but it can be of great value for those living in a water-scarce region
- Need to clarify: for whom? and in what situation?

Always in relative terms

- Value of one good can be quantified only when it is compared to the value of something else
- A cup of coffee can be worth 10 glasses of water or 300 grams of sugar
- Need to specify: relative to what?

Context matters

What is the value of an orange?

- Let $x := (x_1, x_2)$ be a consumption vector of good 1 (apple) and good 2 (orange)
- Say your preference is represented by $U(x_1, x_2)$
- Value of an (additional) orange in units of utility is

$$\Delta U(1) := U(x_1, x_2 + 1) - U(x_1, x_2) \tag{1}$$

which depends on U (for whom) and x (situation)

Marginal utility as the unit value

- More generally, the value of additional Δx_2 units of good 2 can be approximated by

$$\Delta U(\Delta x_2) := \frac{U(x_1, x_2 + \Delta x_2) - U(x_1, x_2)}{\Delta x_2} \Delta x_2 \approx U_2(x) \Delta x_2 \tag{2}$$

Unit of value

In units of utility?

- Recall the unit of utility is not really meaningful
- Same preference represented by $V(x) := 2U(x)$
- When V is used, the value of Δx_2 units of good 2 is

$$V_2(x)\Delta x_2 = 2U_2(x)\Delta x_2 \neq U_2(x)\Delta x_2 \tag{3}$$

Quantifying values relative to numéraire

- Δx_2 units of good 2 is worth $U_2(x)\Delta x_2$ units of utility
- ΔU units of utility is worth $(U_1(x))^{-1}\Delta U$ units of good 1
- Value of Δx_2 units of good 2 relative to good 1 is hence

$$(U_1(x))^{-1}U_2(x)\Delta x_2 = \frac{U_2(x)}{U_1(x)}\Delta x_2, \tag{4}$$

which is independent of the choice of utility function

Generalization

Arbitrary number of goods

- Let $x = (x_1, x_2, \dots, x_N)$ be a consumption bundle
- Consider a person whose preference is given by $U(x)$
- Value of $\Delta x := (\Delta x_1, \Delta x_2, \dots, \Delta x_N)$ in units of good 1 is

$$\frac{U(x+\Delta x) - U(x)}{U_1(x)} \approx \sum_{n=1}^{N} \frac{U_n(x)}{U_1(x)} \Delta x_n \tag{5}$$

Reinterpreting the formula in a dynamic setting

- Let $x = (x_1, x_2, \dots)$ be a consumption plan over time
- Value of $\Delta x := (\Delta x_1, \Delta x_2, \dots)$ in units of today's good is

$$\frac{U(x+\Delta x) - U(x)}{U_1(x)} \approx \sum_{t=1}^{\infty} \frac{U_t(x)}{U_1(x)} \Delta x_t, \tag{6}$$

where $U_t(x)/U_1(x)$ is called the discount factor

Policy evaluation revisited

Value of policy intervention

- Person $i \in I$ currently consuming $x_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,N})$
- Consider a policy which changes the consumption bundle of $i \in I$ by $(\Delta x_{i,1}, \Delta x_{i,2}, \ldots, \Delta x_{i,N}) \in \mathbb{R}^N$
- What is the social value of this policy?

Cost-benefit analysis

- Valuing each $\Delta x_{i,n}$ in the common unit makes inter-goods and inter-personal comparison possible
- Net social value of this policy (in units of good 1) is

$$NV := \sum_{i \in I} NV_i \text{ where } NV_i := \sum_{n=1}^{N} \frac{U_n^i(x_i)}{U_1^i(x_i)} \Delta x_{i,n} \qquad (7)$$

- NV_i is the net value (in units of good 1) for agent i

Example 1

Single-agent two-goods economy

- Preference is represented by $U(c, \bar{l} - l)$
- General consumption good c and leisure time $\bar{l} - l$
- Technology available which transforms Δl units of labor into Δc units of consumption via $\Delta c = f(\Delta l)$

Facilitating longer working time

- Consider a policy which forces her to work for Δl units of time in return for $\Delta c = f(\Delta l)$ units of consumption
- Value of this policy (in units of consumption good) is

$$NV = \frac{U_1(c, \bar{l} - l)}{U_1(c, \bar{l} - l)} f(\Delta l) - \frac{U_2(c, \bar{l} - l)}{U_1(c, \bar{l} - l)} \Delta l, \tag{8}$$

where the first term is benefit while the second is cost

Example 2

Cake-eating problem

- Divide a cake for today's x_1 and tomorrow's x_2
- Preference is represented by $U(x_1, x_2)$
- Specify $U(x_1, x_2) := u(x_1) + \beta u(x_2)$ if you want

Consuming too much?

- Net value of a policy measured in units of today's consumption is called the net present value
- Consider a policy which requires her to save additional fraction $\Delta x_1 > 0$ of cake for tomorrow's consumption
- Net present value of this policy is

$$NPV = -\Delta x_1 + \frac{U_2(x)}{U_1(x)}\Delta x_1 = -\Delta x_1 + \beta\frac{u'(x_2)}{u'(x_1)}\Delta x_1, \quad (9)$$

which is positive if and only if $u'(x_1) < \beta u'(x_2)$

Example 3

Multi-agent economy

- Economy with two agents A and B with two goods
- Preference is represented by $U^i(x_{i,1}, x_{i,2})$ for $i \in \{A, B\}$
- Redistribution $(\Delta x_{A,1}, \Delta x_{A,2}, \Delta x_{B,1}, \Delta x_{B,2})$ is feasible if $\Delta x_{A,l} + \Delta x_{B,l} = 0$ for each $l \in \{1, 2\}$

Social value of redistribution policy

- Value of this redistribution policy (in units of good 1) is

$$NV = \frac{U_2^A(x_A)}{U_1^A(x_A)}\Delta x_{A,2} + \frac{U_2^B(x_B)}{U_1^B(x_B)}\Delta x_{B,2}, \tag{10}$$

which can be positive if and only if

$$\frac{U_2^A(x_A)}{U_1^A(x_A)} \neq \frac{U_2^B(x_B)}{U_1^B(x_B)} \tag{11}$$

Potential Pareto improvement

Decision criteria

- Pareto criterion: policies should be implemented if it achieves Pareto improvement (i.e., $NV_i > 0$ for all i)
- Cost-benefit criterion: policies should be implemented if its net value is positive (i.e., $NV := \sum_i NV_i > 0$)
- Pareto criterion is a strictly stronger requirement

What does the cost-benefit criterion mean?

- If a policy satisfies Pareto criterion, then it satisfies the cost-benefit criterion as well
- Converse is not true in multi-agent settings ($NV_j < 0$ is allowed for some j even if $NV := \sum_i NV_i > 0$)
- If NV is positive, however, then Pareto improvement is potentially possible

Compensation

Kaldor-Hicks compensation criteria

- Kaldor's compensation criterion: policies should be implemented if the gainers can compensate the losers
- Hicks' compensation criterion: policies should not be implemented if the losers can compensate the gainers
- These criteria coincide for 'small' projects

Equivalence to the cost-benefit criterion

- If $NV := \sum_i NV_i > 0$, then there exists $(\delta_i)_{i \in I}$ such that

$$NV_i + \delta_i > 0 \text{ and } \sum_{i \in I} \delta_i = 0 \tag{12}$$

- $(\delta_i)_{i \in I}$ is a compensation vector in units of good 1
- Hence, $NV > 0$ implies that the policy can achieve Pareto improvement if it is jointly implemented with an appropriately designed compensation policy

Example 3 (with compensation)

Redistribution policy

- Suppose $U_2^A/U_1^A > U_2^B/U_1^B$ in Example 3
- Value of redistribution policy $(\Delta x_{A,2}, \Delta x_{B,2}) = (\varepsilon, -\varepsilon)$ is

$$NV = NV_A + NV_B = (U_2^A/U_1^A - U_2^B/U_1^B)\varepsilon > 0 \quad (13)$$

- This policy passes the cost-benefit test, but does not achieve Pareto improvement in itself ($NV_A > 0 > NV_B$)

Compensation

- Consider a compensation policy $(\Delta x_{A,1}, \Delta x_{B,1}) = (-\delta, \delta)$ such that

$$(U_2^A/U_1^A)\varepsilon > \delta > (U_2^B/U_1^B)\varepsilon \quad (14)$$

- Then

$$NV_i + \delta_i > 0 \quad \forall i \in \{A, B\} \quad (15)$$

How do you compute the net social value?

Use market prices, if possible

- Shadow value U_n^i / U_1^i is not observable
- If goods are traded in the market, however,

$$\frac{U_n^i(x_i)}{U_1^i(x_i)} = \frac{p_n}{p_1} \quad \text{for all } i \text{ and } \text{ for all } n, \tag{16}$$

a direct consequence of utility maximization behavior

Computing the net social value

- Net social value (in units of good 1) of Δx is then

$$NV := \sum_{i \in I} \sum_{n=1}^{N} \frac{U_n^i(x_i)}{U_1^i(x_i)} \Delta x_{i,n} = \sum_{n=1}^{N} \frac{p_n}{p_1} \sum_{i \in I} \Delta x_{i,n} \tag{17}$$

- Observe how the distributional issue is circumvented

Cost-benefit analysis in dynamic settings

Same idea

- If there exists a capital market over the time horizon, the discount factor is given by

$$\frac{U_t^i(x_i)}{U_1^i(x_i)} = \prod_{s=2}^{t} \frac{U_s^i(x_i)}{U_{s-1}^i(x_i)} = \prod_{s=2}^{t} \frac{1}{1+r_s} \quad \text{for all } i,t, \tag{18}$$

  where r_s is the real interest rate (from period $s-1$ to s)

- In particular, if $r_t = r$ (constant) for all t, then the net present value of Δx is

$$NPV = \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} \sum_{i \in I} \Delta x_{i,t}, \tag{19}$$

  which suggests that cost and benefit expected in the future should be discounted at the rate of r

Caveats

Non-traded goods

- Cost or benefit of public policies often materializes in the form of non-traded goods (environmental quality, benefit in a far distant future)
- Then the market price is not directly available
- Need to compute the shadow value U_n^i / U_1^i somehow

Price distortion

- Even if the market prices are available, they might not correctly reflect the shadow value
- If there exists an externality, for instance, we often have $U_n^i / U_1^i \neq p_n / p_1$
- Then computing the net social value based on market prices does not provide the correct cost-benefit criterion