

Introductory microeconomics 5

Efficiency

Hiroaki Sakamoto

July 21, 2015

Welfare economics

Primary goals of economics

- Construct a model that can predict the outcome of real-world economic activities (**how things are**)
- Identify the set of “desirable” outcomes in the economy (**how things should be**)
- Design a policy/mechanism that can achieve the “desirable” outcome (**what to do**)

What does “desirable” mean?

- Depends on which criterion we want to use
- Widely-accepted criterion is **Pareto efficiency**
- Another is **equity** (but far more controversial)
- One could argue for yet another

Exchange economy

Division of goods and leisure

- Consider a farm where two workers are producing some agricultural crop
- 4kg of crops are produced and shared by the two
- They can enjoy in total 6 hours of leisure a day

Allocation

- An **allocation** in this economy is a list (vector)

$$a := (x_1, r_1, x_2, r_2) \in \mathbb{R}_+^4, \quad (1)$$

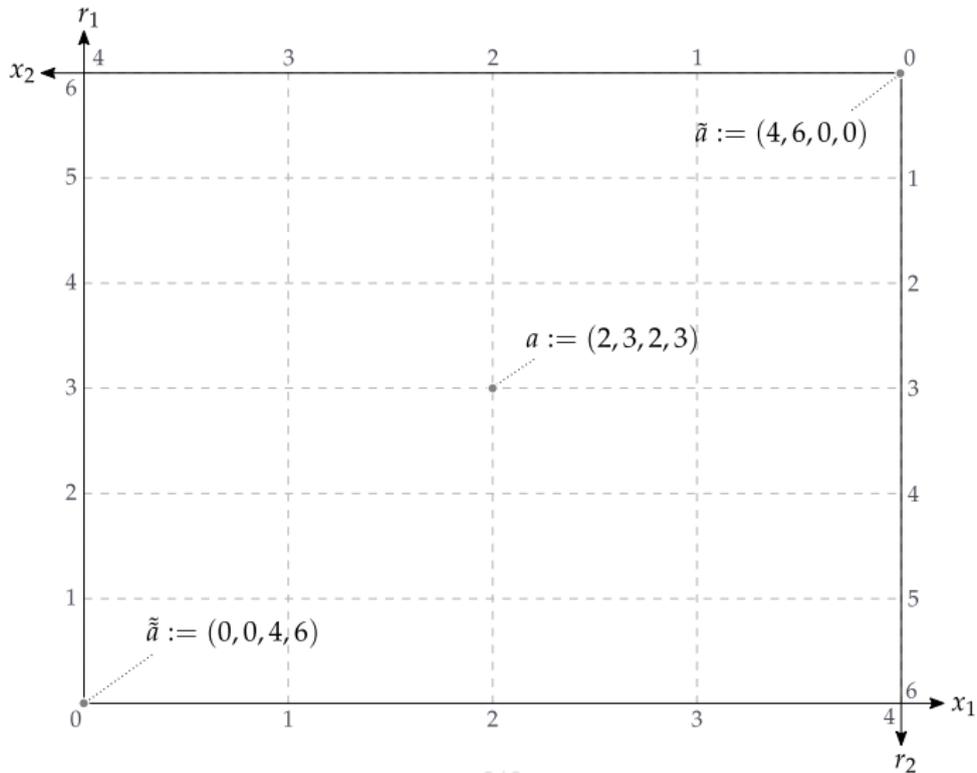
where x_i and r_i are individual i 's crop and leisure

- An allocation a is said to be **feasible** if

$$x_1 + x_2 = 4 \quad \text{and} \quad r_1 + r_2 = 6 \quad (2)$$

- Denote by $A \subseteq \mathbb{R}_+^4$ the set of all feasible allocations

Edgeworth box



Socially desirable allocations

Which allocation is desirable?

- Is an allocation $a := (x_1, r_1, x_2, r_2) = (2, 3, 2, 3)$ desirable?
- What about $\tilde{a} := (\tilde{x}_1, \tilde{r}_1, \tilde{x}_2, \tilde{r}_2) = (2, 2, 2, 4)$?
- Depends on what kind of criterion we would use

Equality and equity

- One could argue that $(2, 3, 2, 3)$ would be the most desirable allocation in terms of **equality**
- But what if, say, individual 2 is physically disabled?
- Wouldn't it be **unfair** to require him to do the same amount of work?
- Perhaps better to choose $(2, 2, 2, 4)$ instead
- Fairness (or **equity**), however, is not easy to define in a universally acceptable way

Efficiency in exchange economy

Pareto dominated allocations

- An allocation $a \in A$ is said to be **Pareto dominated** by another allocation $\tilde{a} \in A$ if

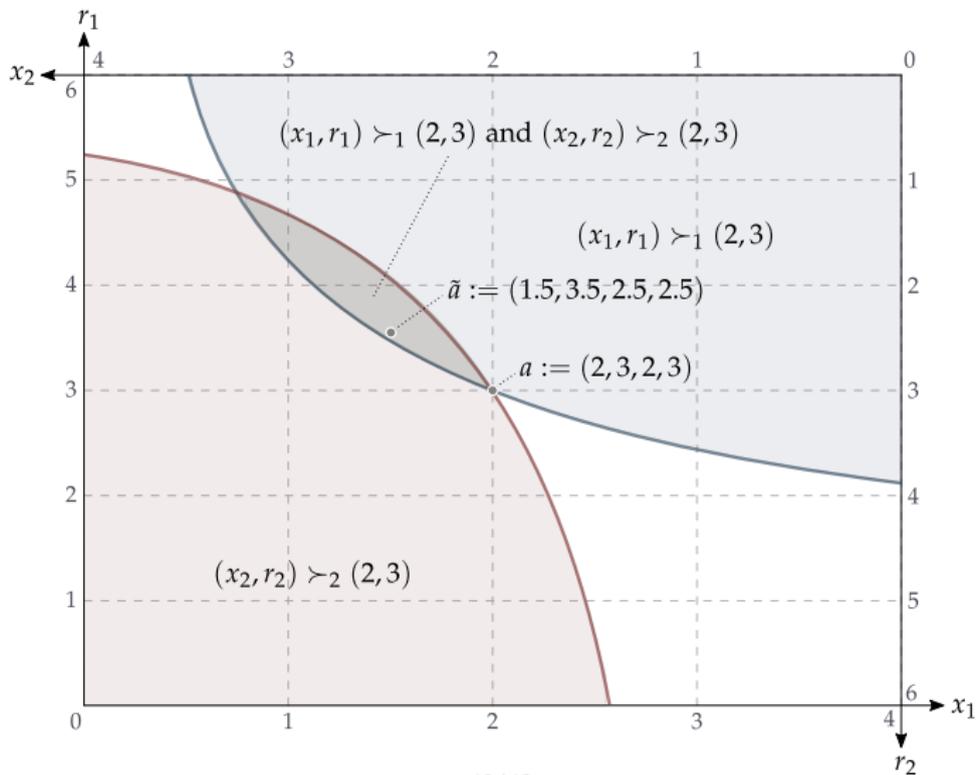
$$(\tilde{x}_i, \tilde{r}_i) \succsim_i (x_i, r_i) \quad \forall i \quad \text{and} \quad (\tilde{x}_i, \tilde{r}_i) \succ_i (x_i, r_i) \quad \exists i \quad (3)$$

- Nobody would disagree that a is **undesirable** then
- What about \tilde{a} ? (could be undesirable since there might be yet another allocation $\tilde{\tilde{a}}$ which Pareto dominates \tilde{a})

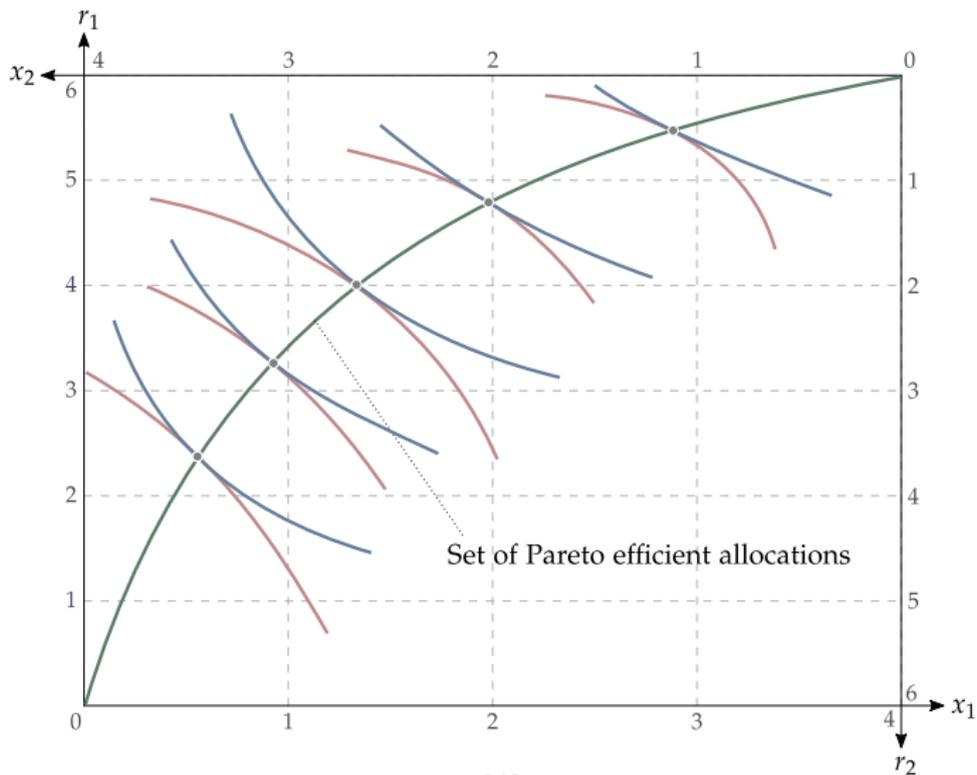
Pareto efficient allocations

- An allocation is said to be **Pareto efficient** if it is not Pareto dominated by any other feasible allocation
- In other words, Pareto efficient allocations are the allocations that are **not clearly undesirable**
- Efficiency does not imply 'best' or 'optimal'

Pareto dominance and improvement



Pareto efficient allocations



Production economy

Potential inefficiency on the production side

- We have so far assumed that $2\bar{z} = 6$ hours of labor is always necessary for producing 4kg of crops
- But what if 4kg of crops can be produced by shorter hours of labor (and hence longer hours of leisure)?
- Moreover, what if 4kg of crops is more than what the individuals actually want?

Pareto efficiency in production economy

- In production economy, we say that an allocation is Pareto efficient if
 - there is no room for improvement in the **production pattern**, AND
 - no room for improvement in the **consumption pattern** of produced goods as well

Example

Production technology

- Put $\bar{z} := 9$ so that 6 hours of leisure (in total) implies $2\bar{z} - 6 = 12$ hours of aggregate labor supply
- Let us say there are two production technologies

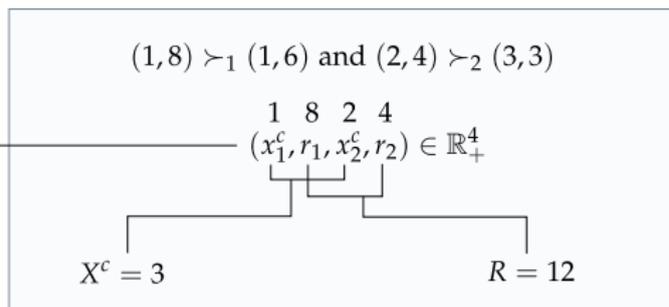
$$Y_1 := \{(4, 2), (7, 3)\}, \quad Y_2 := \{(2, 1), (8, 2)\} \quad (4)$$

- For instance, 4 hours of labor produces 2kg of crops in Y_1 while 2 hours of labor produces 1kg of crops in Y_2

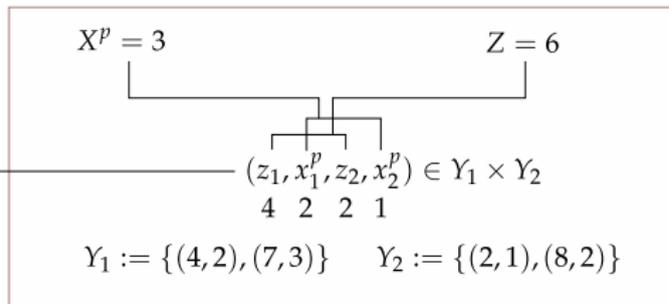
Inefficient production pattern

- Since $(4, 2) \in Y_1$ and $(8, 2) \in Y_2$, 12 hours of aggregate labor (i.e., $2\bar{z} - 12 = 6$ hours of aggregate leisure) can produce 4kg of crops
- But since $(7, 3) \in Y_1$ and $(2, 1) \in Y_2$, the same amount of crops can be produced by 9 hours of aggregate labor by simply reallocating labor from Y_2 to Y_1

Allocations in production economy



$a := (x_1^c, r_1, x_2^c, r_2, z_1, x_1^p, z_2, x_2^p)$ is feasible if $X^c = X^p$ and $2\bar{z} - R = Z$



Generalizing the idea

Allocation and its feasibility

- Consider an economy consisting of
 - I consumers with preference \succsim_i and resource \bar{z}
 - J firms with technology $Y_j \subseteq \mathbb{R}_+^2$
- An **allocation** in this economy is a list

$$a := ((x_i^c, r_i)_{i=1}^I, (z_j, x_j^p)_{j=1}^J) \in \mathbb{R}_+^{2I+2J} \quad (5)$$

such that $(z_j, x_j^p) \in Y_j$ for all $j \in \{1, 2, \dots, J\}$

- An allocation is said to be **feasible** if

$$\sum_i x_i^c = \sum_j x_j^p \text{ and } \sum_i (\bar{z} - r_i) = \sum_j z_j \quad (6)$$

- Each feasible allocation describes **one possible outcome** in the economy

Pareto dominance

Comparing two allocations

- Let $A \subseteq \mathbb{R}_+^{2I+2J}$ be the set of all **feasible** allocations
- Consider two allocations feasible in the economy:

$$a := ((x_i^c, r_i)_{i=1}^I, (z_j, x_j^p)_{j=1}^J) \in A \quad (7)$$

and

$$\tilde{a} := ((\tilde{x}_i^c, \tilde{r}_i)_{i=1}^I, (\tilde{z}_j, \tilde{x}_j^p)_{j=1}^J) \in A \quad (8)$$

- a is said to be **Pareto dominated** by \tilde{a} if

$$(\tilde{x}_i^c, \tilde{r}_i) \succsim_i (x_i^c, r_i) \text{ for all } i \in \{1, 2, \dots, I\} \quad (9)$$

and

$$(\tilde{x}_i^c, \tilde{r}_i) \succ_i (x_i^c, r_i) \text{ for some } i \quad (10)$$

- Someone can be strictly better off without making anybody else worse off

Efficiency

Pareto efficiency

- We say that an allocation is **Pareto efficient** if it is not Pareto dominated by any other feasible allocations
- Economists always use the term efficiency to mean Pareto efficiency

As a criterion of desirable outcome

- If an allocation is not Pareto efficient, you can make everybody happier just by reallocating resources!
- Pareto efficiency is hence **a necessary condition** for an allocation to be “desirable”
- But, quite importantly, **not a sufficient condition**
- Highly unequitable allocations can be Pareto efficient

How can we achieve efficiency?

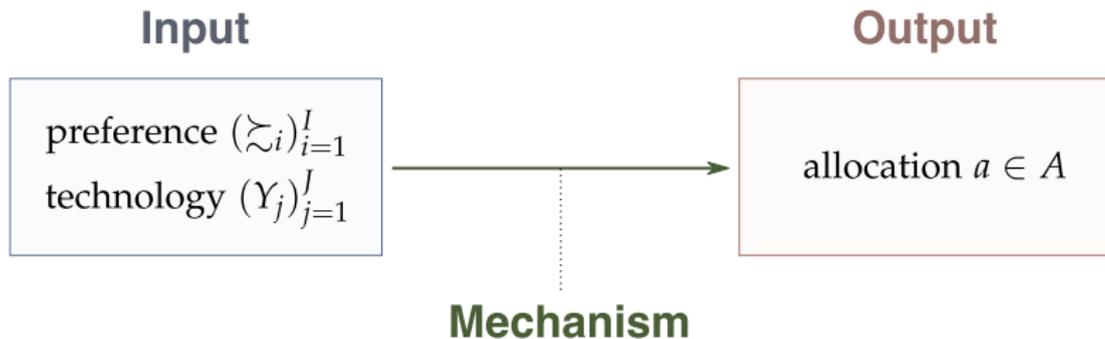
Centralized mechanism

- One obvious mechanism is the planned economy, where the authority controls the resource allocation
- This should work in theory, but not in practice b/c:
 1. huge amount of private information required (in addition, not incentive compatible)
 2. limited computational and bureaucratic capacity
 3. central authority is required to have an incentive to achieve socially desirable outcomes

Decentralized mechanism

- Alternatively, one could use a decentralized mechanism known as **competitive market**
- Competitive market, if properly designed, can achieve efficiency single-handedly

Designing a mechanism



We want to design a mechanism which *always* produces a *desirable* output for *any input*

- *always*: easy to implement, w/o strong assumptions
- *desirable*: efficient, at least
- *any input*: widely applicable and highly scalable

Fundamental theorem

First fundamental theorem of welfare economics

- Any competitive equilibrium is Pareto efficient
- That is, the outcome under competitive market always achieves efficiency in any economy
- No matter how many consumers, how many firms, and how many markets exist in the economy
- Almost no assumption on preference and technology

Proof (by contradiction)

- Suppose, by way of contradiction, that the competitive equilibrium is not Pareto efficient
- Then there exists another 'more efficient' allocation
- But we can show that such an allocation is infeasible
- Therefore, the competitive equilibrium must be Pareto efficient

Implications of fundamental theorem

Formalization of Adam Smith's invisible hand

- Decisions of consumers and firms are made purely based on their self interest
- Nobody intends to achieve socially desirable outcomes
- Individuals' efforts to pursue their own interest automagically benefit society in competitive markets

Market as an efficient resource-allocation mechanism

- One could see market as a mechanism under which resources are reallocated among economic agents
- Fundamental theorem clarifies the function of market and demonstrates its advantage
- Informationally less demanding and highly scalable
- We do not even need to know in advance what the efficient allocation looks like

Caveats

Market failure

- Market, if it is not well functioning, can not achieve Pareto efficiency
- Two important cases when market fails:
 - imperfect competition (e.g., monopoly)
 - externalities (e.g., pollution)

Equity

- First fundamental theorem is completely silent on equity (distributional aspect of resource allocation)
- Even when it is well functioning, resource allocation under competitive market can be highly unequitable
- Distributional issues can be (partially) dealt with by the [second fundamental theorem of welfare economics](#), which will be covered in more advanced courses

Review: competitive market

Firms are price-takers

- Input-output combination chosen from their production set in such a way that the profit is maximized
- Crucial assumption: price is **taken as given**
- This is reasonable when there are many similar firms competing with each other in the market
- Price is **in effect** given in competitive markets

Flip side of the same coin

- Firms can sell as many goods as they want **for a given price** (i.e., without affecting the price)
- Reasonable when each firm is sufficiently small
- Any adjustment in their behavior (supply of goods) does not affect the market price

Monopolistic market

Monopolist

- What if there is only one firm (i.e., **monopolist**) operating in the market?
- Then the monopolist can freely choose the price
- On the flip side, the monopolist cannot take the price given when increasing its supply

Price-quantity trade-off

- Price can be freely chosen, but then the quantity it can sell in the market is automatically determined
 - Alternatively, the monopolist can choose the quantity of supply, but then the price is automatically determined
 - Increasing the price will decrease the demand
- likely to end up with **overpricing** (and **undersupply**) and therefore inefficiency follows

Externality

Welfare theorem reconsidered

- Welfare theorem shows that efficiency will be achieved at competitive equilibrium
- This is not necessarily the case in the presence of what we call externalities
- Externality is one primary reason for governmental intervention being justified

Definition

- We say that there is an externality if an action of one agent directly affects other agents in the economy
- By 'directly,' we mean 'not through a change of price'
- In other words, an externality is an interaction among agents that is external to the market

Real-world examples

Negative externalities

- Neighbor's consumption of loud music late at night
 - Water pollution due to an upstream factory
 - Individual's abuse of antibiotics (which has the risk of making bacteria resistant to antibiotics)
 - Keeping up with the Jones (positional externality)
- over-consumption or -production of undesirable goods

Positive externalities

- Maintaining a garden that is attractive to neighbors
 - Pleasant smell of baking bread at a local bakery
 - Becoming a member of social network sites or learning languages (network externality)
- under-consumption or -production of desirable goods