Introductory microeconomics 4
Equilibrium

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Markets and economy

Markets

- We are now ready to describe what we call the market in its simplest form
- Market is a ‘place’ where goods and production factor are traded between consumers and firms
- Market exists for each good and each production factor

Economy

- Our goal is to describe the entire economy
- First focus on each market and then combine them
- Keep in mind that markets interact with each other:
  - how many goods you buy in goods market depends on how much income you earn in labor market
  - how much income you earn in labor market depends on how many goods you buy in goods market

Consumers

Consumers

- Consider an economy with $I \in \mathbb{N}$ consumers
- Assume that preference is represented by a utility function $U^i(x_i, r_i)$ for each $i \in \{1, 2, \ldots, I\}$
- Here $r_i := \bar{z} - z_i$ is leisure time of consumer i

Demand function

- Demand functions of consumer i are given by

$$(x_i^d(p, w, m), r_i^d(p, w, m)) \in \arg\max_{(x_i, r_i) \in B_i} U^i(x_i, r_i) \quad (1)$$

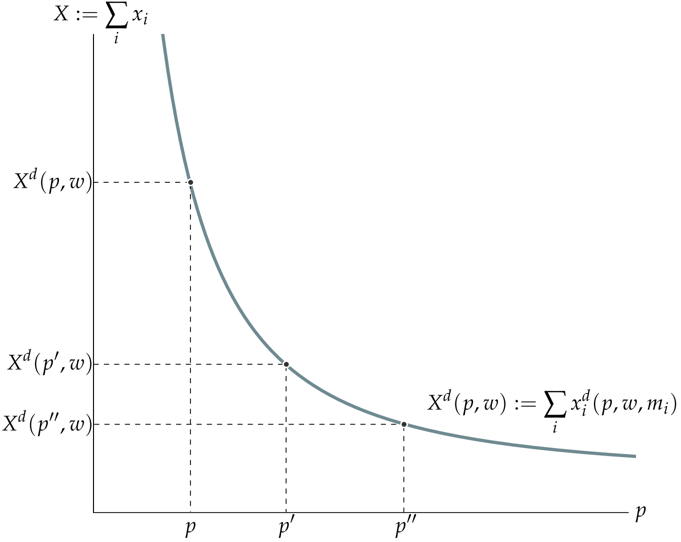
where

$$B_i := \{(x_i, r_i) \in \mathbb{R}_+^2 \mid px_i + wr_i = M_i := w\bar{z} + m_i\}$$

- Aggregate demand function for good x is then

$$X^d(p, w) := \sum_{i=1}^{I} x_i^d(p, w, m_i) \quad (2)$$

Aggregate demand function



Firms

Firms

- Suppose that there are $J \in \mathbb{N}$ firms producing good x
- Let $Y_j \subseteq \mathbb{R}^2_+$ be the production set of firm $j \in \{1, 2, \ldots, J\}$
- Assume Y_j is represented by a cost function c_j

Supply function

- Factor demand and supply function of firm j are

$$(z_j^d(w,p), x_j^s(w,p)) \in \arg\max_{(z,x)\in Y_j} \{px - wz\} \quad (3)$$

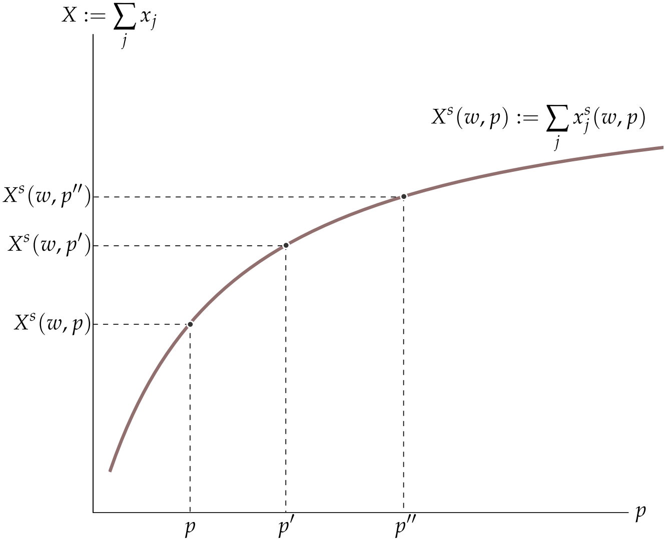
or, equivalently,

$$x_j^s(w,p) \in \arg\max_{x\in\mathbb{R}_+} \{px - c_j(x)\} \quad (4)$$

- Aggregate supply function is then

$$X^s(w,p) := \sum_{j=1}^{J} x_j^s(w,p) \quad (5)$$

Aggregate supply function



(Partial) equilibrium

What is equilibrium?

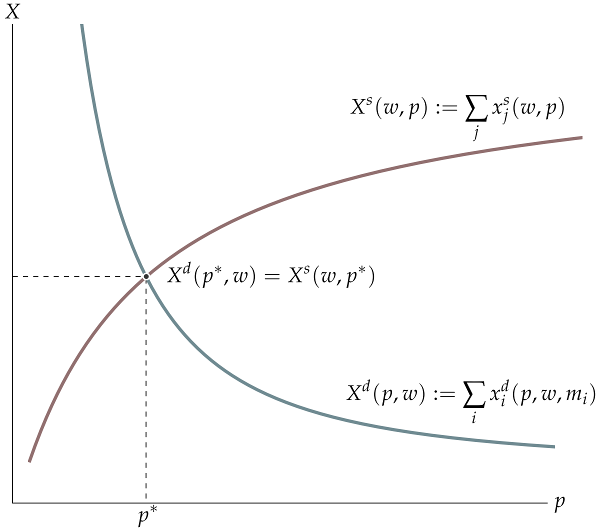
- Equilibrium is the state of economy/market which economists think will materialize at the end of the day
- When we talk about equilibrium, we basically talk about our prediction (our answer to the question "what would happen in this economy/market?")

Equilibrium in goods market

- In the competitive market, the aggregate demand and supply will eventually coincide
- It is hence natural to define equilibrium in the market of good x as the state where $X^d = X^s$
- A price level p^* is called the equilibrium price if

$$X^d(p^*, w) = X^s(w, p^*) \tag{6}$$

Equilibrium in goods market



Example

Setup

- Preference: $U^i(x_i, r_i) := x_i^{1/2} r_i^{1/2}$ for all $i \in \{1, \ldots, I\}$
- Technology: $c_j(x_j) := w x_j^2$ for all $j \in \{1, \ldots, J\}$

Equilibrium in goods market

- Demand: $x_i^d(p, w) = M_i/(2p)$, where $M_i := w\bar{z} + m_i$
- Supply: $x_j^s(w, p) = p/(2w)$
- If p^* is the equilibrium price,

$$X^d(p^*, w) = X^s(w, p^*) \iff \frac{\sum_i M_i}{2p^*} = \frac{Jp^*}{2w}, \tag{7}$$

which implies

$$p^* = \left(\frac{w}{J} \sum_{i=1}^{I} M_i\right)^{1/2} = \left(\frac{w}{J}\left(Iw\bar{z} + \sum_{i=1}^{I} m_i\right)\right)^{1/2} \tag{8}$$

Equilibrium in labor market

Labor market

- Aggregate labor supply is

$$Z^s(p,w) := \sum_{i=1}^{I} z_i^s(p,w) = \sum_{i=1}^{I}(\bar{z} - r_i^d(p,w)) \tag{9}$$

- Aggregate labor demand, on the other hand, is

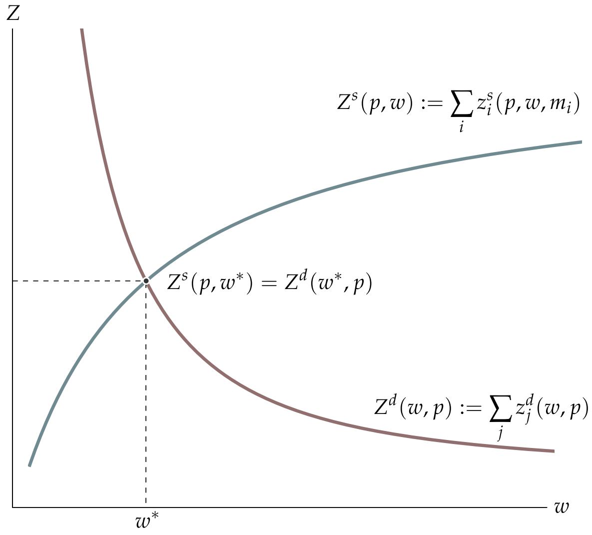
$$Z^d(w,p) := \sum_{j=1}^{J} z_j^d(w,p) \tag{10}$$

Equilibrium in labor market

- As in goods market, we define equilibrium in labor market as the state where $Z^d = Z^s$
- A price level w^* is called the equilibrium factor price or equilibrium wage rate if

$$Z^d(w^*,p) = Z^s(p,w^*) \tag{11}$$

Equilibrium in labor market



Example

Setup

- Preference: $U^i(x_i, r_i) := x_i^{1/2} r_i^{1/2}$ for all $i \in \{1, \ldots, I\}$
- Technology: $f_j(z_j) := z_j^{1/2}$ for all $j \in \{1, \ldots, J\}$

Equilibrium in labor market

- Supply: $z_i^s(p, w) = \bar{z} - r_i^d(p, w) = (w\bar{z} - m_i)/(2w)$
- Demand: $z_j^d(w, p) = (p/(2w))^2$
- If w^* is the equilibrium wage rate,

$$Z^s(p, w^*) = Z^d(w^*, p) \iff \frac{Iw^*\bar{z} - \sum_i m_i}{2w^*} = J\left(\frac{p}{2w^*}\right)^2, \quad (12)$$

which implies

$$w^* = \frac{\sum_i m_i}{2I\bar{z}} + \left(\frac{Jp^2}{2I\bar{z}} + \left(\frac{\sum_i m_i}{2I\bar{z}}\right)^2\right)^{1/2} \quad (13)$$

Missing piece: distribution of profits

Who is behind the firms?

- Behind each firm are consumers who own the firm
- Maximized profits of firms will be eventually distributed among the shareholders

Modeling ownership structure

- Consumer i owns a share $\theta_{i,j} \in [0,1]$ of firm j so that she can claim fraction $\theta_{i,j}$ of firm j's profit π_j:

$$m_i = \sum_{j=1}^{J} \theta_{i,j}\pi_j \tag{14}$$

- Ownership structure may be represented by the list

$$((\theta_{1,j})_{j=1}^{J}, (\theta_{2,j})_{j=1}^{J}, \ldots, (\theta_{I,j})_{j=1}^{J}) \in \mathbb{R}_+^{I\times J} \tag{15}$$

- Notice $\sum_{i=1}^{I} \theta_{i,j} = 1$ for each j and thus $\sum_i m_i = \sum_j \pi_j$

Example

Maximized profit

- Technology: $c_j(x_j) := wx_j^2$ for all $j \in \{1, \ldots, J\}$
- Given w and p, the optimal level of supply is

$$c_j'(x_j^*) = p \iff x_j^* = p/(2w) \tag{16}$$

- Maximized profit of firm j is hence

$$\pi_j := px_j^* - c_j(x_j^*) = p^2/(4w) \tag{17}$$

Distribution of profits

- Non-labor income m_i of consumer i is then

$$m_i = \sum_j \theta_{i,j}\pi_j = \sum_j \theta_{i,j}p^2/(4w) \tag{18}$$

- Therefore

$$M_i = w\bar{z} + m_i = w\bar{z} + \sum_j \theta_{i,j}p^2/(4w) \tag{19}$$

Example: everything combined

Setup

- Preference: $U^i(x_i, r_i) := x_i^{1/2} r_i^{1/2}$ for all $i \in \{1, \ldots, I\}$
- Technology: $c_j(x_j) := w x_j^2$ for all $j \in \{1, \ldots, J\}$
- Ownership: $((\theta_{1,j})_{j=1}^J, (\theta_{2,j})_{j=1}^J, \ldots, (\theta_{I,j})_{j=1}^J)$

Aggregate demand and supply in each market

- Goods market:

$$X^d(p, w) = \frac{I\bar{z}}{2}\frac{w}{p} + \sum_i \sum_j \theta_{i,j} \frac{p}{8w}, \; X^s(w, p) = \frac{Jp}{2w} \tag{20}$$

- Labor market:

$$Z^s(p, w) = \frac{I\bar{z}}{2} - \sum_i \sum_j \theta_{i,j} \frac{p^2}{8w^2}, \; Z^d(w, p) = \frac{Jp^2}{4w^2} \tag{21}$$

Equilibrium

Equilibrium in the entire economy

- Our model economy now consists of two markets: goods market and labor market
- We define equilibrium in this model economy as the state where $X^d = X^s$ and $Z^s = Z^d$
- A list (p^*, w^*) is called the equilibrium price vector if

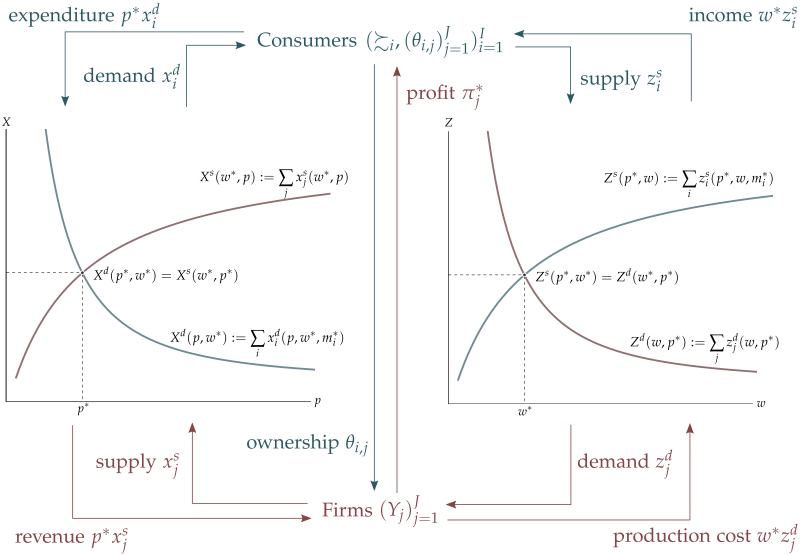
$$X^d(p^*, w^*) = X^s(w^*, p^*) \text{ and } Z^s(p^*, w^*) = Z^d(w^*, p^*)$$

- The list

$$((x_i^d(p^*, w^*), z_i^s(p^*, w^*))_{i=1}^{I}, (x_j^s(w^*, p^*), z_j^d(w^*, p^*))_{j=1}^{J})$$

  is called the equilibrium allocation
- Equilibrium allocation is the outcome that we think will materialize in this economy

Equilibrium in the entire economy



Example

Setup

- Preference: $U^i(x_i, r_i) := x_i^{1/2} r_i^{1/2}$ for all $i \in \{1, \ldots, I\}$
- Technology: $c_j(x_j) := w x_j^2$ for all $j \in \{1, \ldots, J\}$

Equilibrium in the entire economy

- If (p^*, w^*) is an equilibrium price vector,

$$\begin{aligned} X^d(p^*, w^*) &= X^s(w^*, p^*) \\ Z^s(p^*, w^*) &= Z^d(w^*; p^*) \end{aligned} \iff \frac{p^*}{w^*} = \left(\frac{4I\bar{z}}{3J}\right)^{1/2} \qquad (22)$$

- Equilibrium allocation is hence

$$x_i^d(p^*, w^*) = \left(\frac{3J}{4I} + \frac{\sum_j \theta_{i,j}}{4}\right)\left(\frac{I\bar{z}}{3J}\right)^{\frac{1}{2}}, \quad x_j^s(w^*, p^*) = \left(\frac{I\bar{z}}{3J}\right)^{\frac{1}{2}}$$

$$z_i^s(p^*, w^*) = \tfrac{1}{2}\bar{z} - \textstyle\sum_j \theta_{i,j} \frac{I\bar{z}}{6J}, \quad z_i^d(w^*, p^*) = \frac{I\bar{z}}{3J}$$