Introductory microeconomics 3
Production factor market

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July 7, 2015

Review: model of consumers

Demand function

- Consider a consumer whose preference is represented by a utility function $U(x_1, x_2)$
- Demand functions are given by

  $$(x_1^d(p_1, p_2, M), x_2^d(p_1, p_2, M)) \in \arg\max_{(x_1,x_2)\in B} U(x_1, x_2)$$

  where $B := \{(x_1, x_2) \in \mathbb{R}_+^2 \mid p_1x_1 + p_2x_2 = M\}$
- Solution of the utility maximization problem (UMP) can be found by solving

  $$\frac{U_1(x_1^*, x_2^*)}{U_2(x_1^*, x_2^*)} = \frac{p_1}{p_2} \text{ and } p_1x_1^* + p_2x_2^* = M \tag{1}$$

  for (x_1^*, x_2^*)
- But wait... where does the income M come from?

Review: model of firms

Supply and factor demand function

- Consider a firm whose technology is characterized by a production set $Y \subseteq \mathbb{R}^2_+$
- Factor demand and supply function are given by

$$(z^d(w,p), x^s(w,p)) \in \arg\max_{(z,x)\in Y}\{px - wz\} \quad (2)$$

- Assume Y is represented by a production function $f : \mathbb{R}_+ \to \mathbb{R}_+$ as well as a cost function $c : \mathbb{R}_+ \to \mathbb{R}_+$
- Then the solution (x^*, z^*) of PMP can be found by

$$pf'(z^*) = w \text{ and } x^* = f(z^*) \quad (3)$$

or

$$p = c'(x^*) \text{ and } z^* = c(x^*)/w \quad (4)$$

- But wait... where does the labor z come from?

Production factor market

Supply side of the factor market

- Firms decide how much production factor they demand, which in turn need to be supplied somehow
- Typically, production factors are supplied by consumers
- In other words, consumers decide how much labor they supply, as well as how many goods they demand

Simplest model of labor-leisure choice

- Denote by $\bar{z}$ the total amount of time available (say, $\bar{z} := 24$ hours a day)
- If they work for $z \leq \bar{z}$ hours, they would earn labor income wz, which can be used for buying good x
- But then they only have $\bar{z} - z$ hours of leisure time
- Consumers choose a pair $(x, r) \in X$ of consumption good x and leisure time $r := \bar{z} - z$

Labor supply

Budget set

- Letting m be non-labor income (if any), the budget set is

$$B = \{(x, r) \in \mathbb{R}^2_+ \mid px = w(\bar{z} - r) + m \text{ and } r \leq \bar{z}\}, \qquad (5)$$

- May be rewritten in a more familiar form as

$$B = \{(x, r) \in \mathbb{R}^2_+ \mid px + wr = M \text{ and } r \leq \bar{z}\}, \qquad (6)$$

where $M := w\bar{z} + m$

Labor supply function

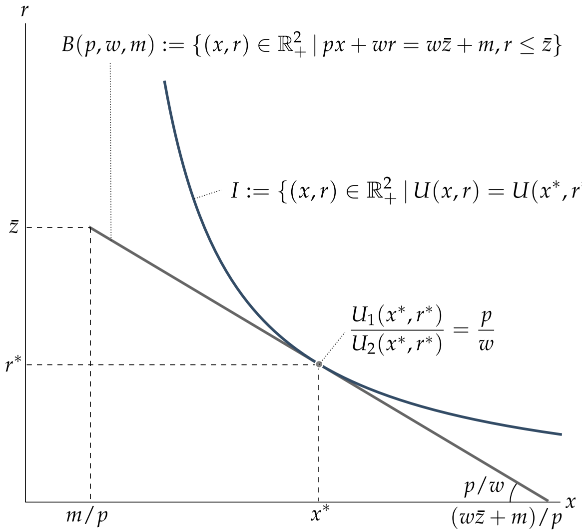
- Demand functions (for good x and leisure r):

$$(x^d(p, w, m), r^d(p, w, m)) \in \arg\max_{(x,r)\in B} U(x, r)$$

- Labor supply function is then

$$z^s(p, w, m) := \bar{z} - r^d(p, w, m) \qquad (7)$$

Labor-leisure choice



Example

Deriving labor supply function

- Suppose that preference about consumption good x and leisure time $r := \bar{z} - z$ is represented by

$$U(x, r) := x^{1/2} r^{1/2} \tag{8}$$

- Recall that the budget constraint is

$$px + wr = M := w\bar{z} + m \tag{9}$$

- If (x^*, r^*) is a solution of UMP,

$$\frac{U(x^*, r^*)}{U_2(x^*, r^*)} = \frac{p}{w} \overset{\text{with (9)}}{\Longrightarrow} (x^*, r^*) = \left(\frac{M}{2p}, \frac{M}{2w}\right) \tag{10}$$

- Hence, the labor supply function is

$$z^s(p, w, m) := \bar{z} - r^d(p, w, m) = \bar{z} - \frac{M}{2w} = \frac{\bar{z}}{2} - \frac{m}{2w} \tag{11}$$