Introductory microeconomics 2
Model of firms

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June 30, 2015

Supply side of the market

Producers

- Consumers decide how many goods they demand, which in turn need to be supplied somehow
- Typically, goods are supplied by producers (those who know how to produce goods)
- Producers themselves are decision makers

What do they decide on?

- Producers decide:
  - how many goods they produce
  - how they produce the goods (potentially, there are many ways of producing the same output)
- In other words, producers choose a particular combination of input and output among technically feasible input-output combinations

Modeling decision making of producers

Model of producers

- How should we model decision making of producers?
- Economists assume: given a set of technically feasible production plans, producers choose the one they like the best
- Just like the model of consumers

Formalizing the idea

- Formally describing this model (by math) requires some clarification
- Need to clarify:
  - technically feasible production plans (alternatives)
  - choose the one they like the best (decision criteria)
- Let us examine these points one by one

Set of alternatives

Description of technology

- Consider a tomato farmer who can either
  - use 1 hour of labor and produce 1 ton of tomatoes,
  - use 4 hours of labor and produce 2 tons of tomatoes,
  - use 9 hours of labor and produce 3 tons of tomatoes
- This list describes the technology of the tomato farmer

Production set

- Set of alternatives for this farmer is hence

$$Y := \{(0,0),(1,1),(4,2),(9,3)\}, \tag{1}$$

  which is called the production (possibility) set
- A mathematical description of producer's technology
- A combination (z,x) of input z and output x is feasible if $(z,x) \in Y$, but not feasible if $(z,x) \notin Y$

Decision criteria

Profit maximization

- One could think of various decision criteria for producers (for-profit or non-profit)
- For the moment, we focus on those producers who try to maximize their profit
- This type of producers are usually called firms

Profit

- Profits are defined as revenue minus cost
- Let (w, p) be the prices of input and output, respectively
- If a combination $(z, x) \in Y$ of input z and output x is chosen, then the profit π is

$$\pi = px - wz, \tag{2}$$

where px is the revenue and wz is the cost

Formal model of firms

Profit maximization problem

- Consider a firm whose technology is described by a production possibility set Y
- Given a price vector (w, p), this firm will choose $(z^*, x^*) \in Y$ such that

$$px^* - wz^* \geq px - wz \quad \forall (z, x) \in Y \tag{3}$$

- We call this profit maximization problem (PMP)

Supply function

- Easy to see that the solution (z^*, x^*) of PMP depends on the price vector (w, p)
- Solution x^*, when seen as a function of (w, p), is called the supply function (denoted by $x^s(w, p)$)
- We call z^* the factor demand function ($z^d(w, p)$)

Example

PMP of a tomato farmer

- Consider a tomato farmer whose production set is

$$Y := \{(0,0),(1,1),(4,2),(9,3)\} \tag{4}$$

- Suppose $(w,p) = (1,4)$
- Then $(z^*,x^*) = (4,2)$ because

$$px^* - wz^* = 4 \geq px - wz \quad \forall (z,x) \in Y \tag{5}$$

Changes in price imply changes in supply

- Now suppose that prices change to $(w,p) = (2,12)$
- Then $(z^*,x^*) = (9,3)$ because

$$px^* - wz^* = 18 \geq px - wz \quad \forall (z,x) \in Y \tag{6}$$

- $(w,p) = (1,4) \to (2,12) \Rightarrow (z^*,x^*) = (4,2) \to (9,3)$

More explicit characterization

Characterizing supply function

- We want to characterize the solution of PMP (supply function in particular) more explicitly
- How the supply function responds to changes in price
- Need to reformulate PMP a little bit

Functional representations of technology

- This is done by considering two different ways of characterizing technology
- Production function: how much output $f(z)$ they can produce for a given level z of input
- Cost function: how much they can minimize the cost $c(x)$ for producing a given level x of output
- These functions can be seen as two alternative representations of the same technology

Describing technology by functions (1)

Production function

- Consider a tomato farmer whose production set is

$$Y := \{(0,0),(1,1),(4,2),(9,3)\} \tag{7}$$

- This technology can be represented by a function

$$f(z) := z^{1/2} \quad \forall z \in Z := \{0,1,4,9\} \tag{8}$$

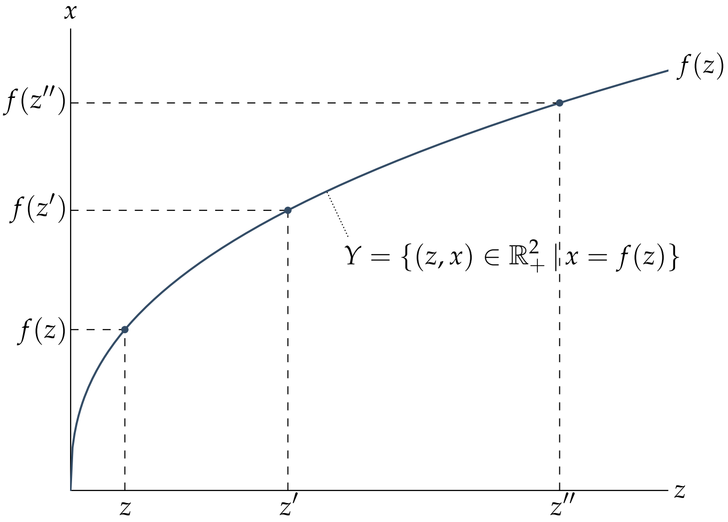
- Observe

$$(z,x) \in Y \iff x = f(z) \tag{9}$$

More general case

- Adjust the level z of inputs in a more flexible way
- Set Z of possible levels of input is then $\mathbb{R}_+$
- Production function is then $f : \mathbb{R}_+ \to \mathbb{R}_+$
- Production set Y associated with f is a subset of $\mathbb{R}^2_+$

Production function



Describing technology by functions (2)

Cost function

- Consider a tomato farmer whose production set is

$$Y := \{(0,0),(1,1),(4,2),(9,3)\} \tag{10}$$

- This technology can be represented by a function

$$c(x) := wx^2 \tag{11}$$

- To produce x, the amount of input required is $z = x^2$
- The cost of producing x is hence $wz = wx^2$

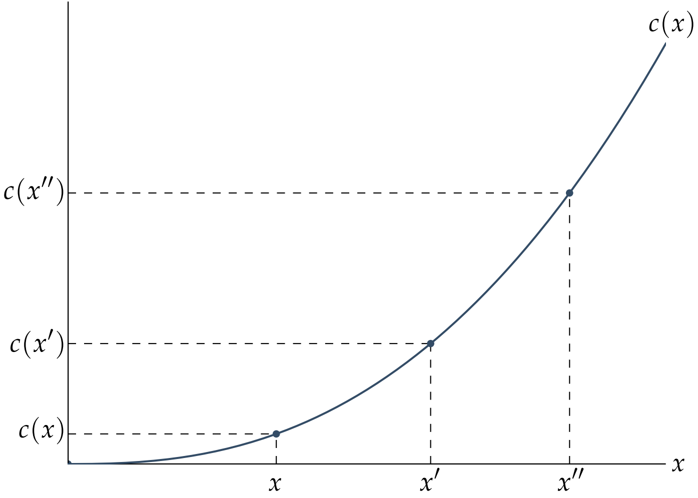
Deriving cost function from production function

- Given a production function $f(z)$, the cost function is

$$c(x) = wf^{-1}(x), \tag{12}$$

where f^{-1} is the inverse function of f

Cost function



Profit maximization problem revisited

Using production function

- Profit maximization problem (3) may be written as

$$z^* \in \arg\max_{z \in \mathbb{R}_+} \{pf(z) - wz\} \tag{13}$$

  and $x^* = f(z^*)$
- Find the optimal level z^* of input first, then compute the associated level x^* of supply

Using cost function

- Alternatively, PMP (3) may be written as

$$x^* \in \arg\max_{x \in \mathbb{R}_+} \{px - c(x)\} \tag{14}$$

  and $z^* = c(x^*)/w$
- Find the optimal level x^* of output, which in turn determines the associated level z^* of input

Example (production function approach)

Deriving supply function

- Production function: $f(z) := z^{2/5}$
- Find (z^*, x^*) such that

$$z^* \in \arg\max_{z \in \mathbb{R}_+} \{pf(z) - wz\} \text{ and } x^* = f(z^*) \tag{15}$$

- If z^* is a solution of this problem, then

$$(pf(z^*) - wz^*)' = 0 \iff pf'(z^*) = w, \tag{16}$$

which means

$$\frac{2p}{5}(z^*)^{-3/5} = w \iff z^d(w,p) := z^* = \left(\frac{2p}{5w}\right)^{5/3} \tag{17}$$

- Supply function is hence

$$x^s(w,p) := x^* = f(z^*) = \left(\frac{2p}{5w}\right)^{2/3} \tag{18}$$

Example (cost function approach)

Deriving supply function

- Cost function: $c(x) := wx^{5/2}$
- Find (z^*, x^*) such that

$$x^* \in \arg\max_{x \in \mathbb{R}_+} \{px - c(x)\} \text{ and } z^* = c(x^*)/w \quad (19)$$

- If x^* is a solution of this problem, then

$$(px^* - c(x^*))' = 0 \iff p = c'(x^*), \quad (20)$$

which means

$$p = \frac{5}{2} w (x^*)^{3/2} \iff x^s(w,p) := x^* = \left(\frac{2p}{5w}\right)^{2/3} \quad (21)$$

- Factor demand function is hence

$$z^d(w,p) := z^* = c(x^*)/w = \left(\frac{2p}{5w}\right)^{5/3} \quad (22)$$

More on cost function approach

Marginal cost

- Derivative $c'(x)$ of cost function is called the marginal cost, which itself is a function of x
- Observe that the level x^* of supply always satisfies

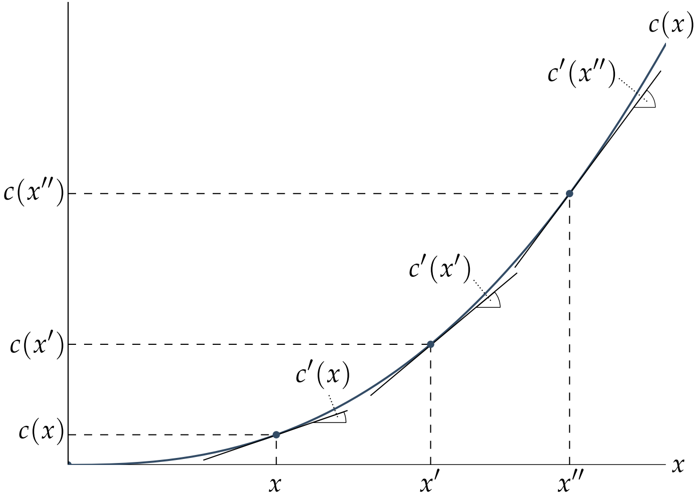
$$c'(x^*) = p \tag{23}$$

- In other words, the supply is determined in such a way that the marginal cost is equalized to the price

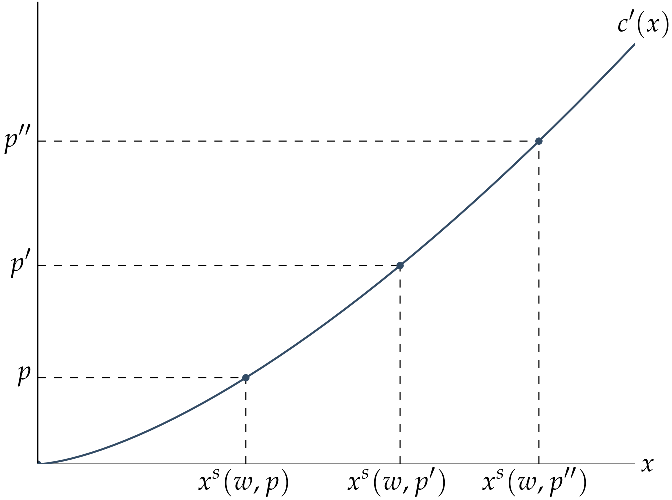
Marginal cost function and supply function

- Supply function can be derived from cost function
- In fact, the marginal cost function $c'(x)$ itself is the (inverse) supply function (b/c $x^* = c'^{-1}(p)$)
- This can be easily seen by drawing figures

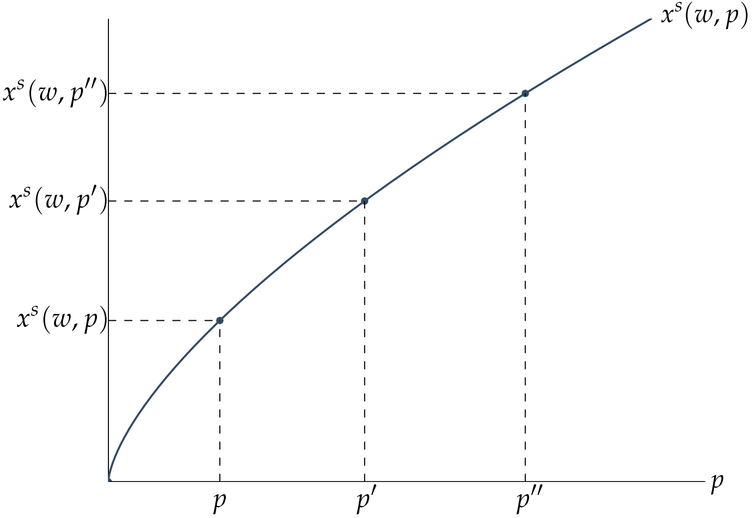
Marginal cost



Marginal cost and supply



Supply function



Exercise

Question 1

- Consider a firm whose technology is characterized by a production function

$$f(z) := z^{1/3} \tag{24}$$

- Denote by w, p the price of production factor and product, respectively
- Derive the supply function $x^s(w, p)$ of this firm

Question 2

- Consider a firm whose technology is characterized by a cost function

$$c(x) := wx^3 \tag{25}$$

- Derive the supply function $x^s(w, p)$ of this firm