Public bads, heterogeneous beliefs, and the value of information

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Plan of talk

Background

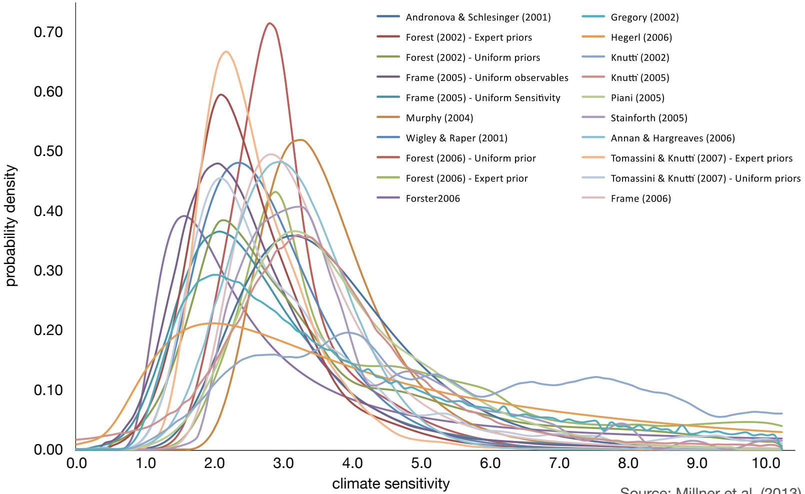
Climate change

- Typical example of public bads
- Studied intensively in environmental/public economics
- Missing in the literature are:
  - ambiguity in negative externality
  - highly heterogeneous beliefs of players
  - role of public information

Ambiguity

- Climate sensitivity is inherently uncertain
- Estimated objective risks in scientific studies not in agreement with each other
- We know climate change is a risk, but not sure how risky

Ambiguity in climate science



Source: Millner et al. (2013)

Heterogeneous beliefs

Subjectivity

- Lack of clear-cut consensus in science
- Interpretation of the proposed risks is subjective
- Disagreements among players allowed

Heterogeneity

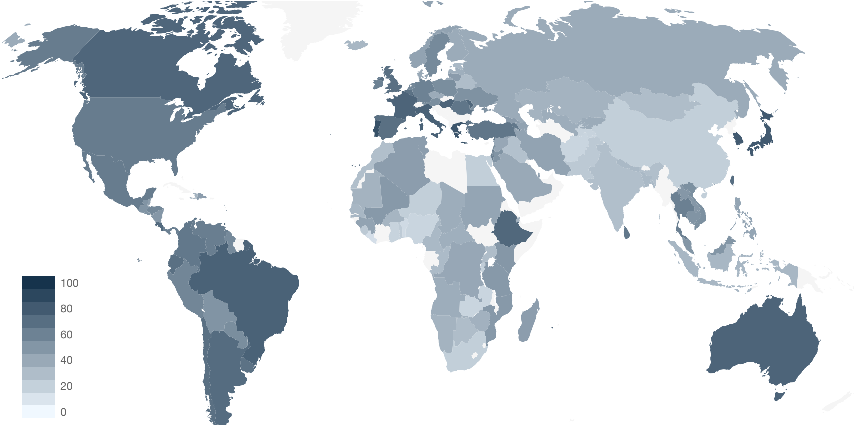
- In fact, people's perceptions significantly vary:

	awareness	*human induced*	*perceived as threat*
France	93%	63%	75%
China	62%	58%	21%
USA	97%	49%	63%
Japan	99%	91%	80%
Russia	85%	52%	39%

Source: *Climate change opinion by country* (Gallup Poll, 2009)

Wide variation of risk perception

Climate change percerived as a threat (%)



Source: Pelham (2009)

Belief and public information

Do heterogeneous beliefs matter?

- Most likely end up with uncoordinated actions
- Optimists abate too little while pessimists too much
- Source of inefficiency
- Of a different kind, on top of the externality

Public information might help

- Reshapes people's posteriors:
  - rationalization based on new information
  - convergence facilitated
- IPCC assessment reports, updated every 5 years or so
- One might say the value of information is positive

$\rightarrow$ Is it always the case? If not, in what condition?

Model

Basic game

- $n \geq 2$ identical players
- Consumption x_i of player i is determined by

$$x_i = \bar{y} - D(E; \beta) - C(a_i), \tag{1}$$

  where
  - $\bar{y}$ is exogenous output, causing emission $\bar{e} := e(\bar{y})$
  - a_i is abatement so that the net emission is $\bar{e} - a_i$
  - $E := \sum_i \bar{e} - \sum_i a_i$, the aggregate net emission
  - D is damage, increasing and convex in E
  - C is abatement cost, increasing and convex in a_i
- (Marginal) damage is increasing in parameter β:

$$\partial D / \partial \beta > 0 \quad \text{and} \quad \partial D' / \partial \beta \geq 0 \tag{2}$$

Uncertainty

Uncertain parameter

- Value of β is unknown with support $B \subseteq \mathbb{R}$
- If the density function $f \in \Delta(B)$ is known, the utility is

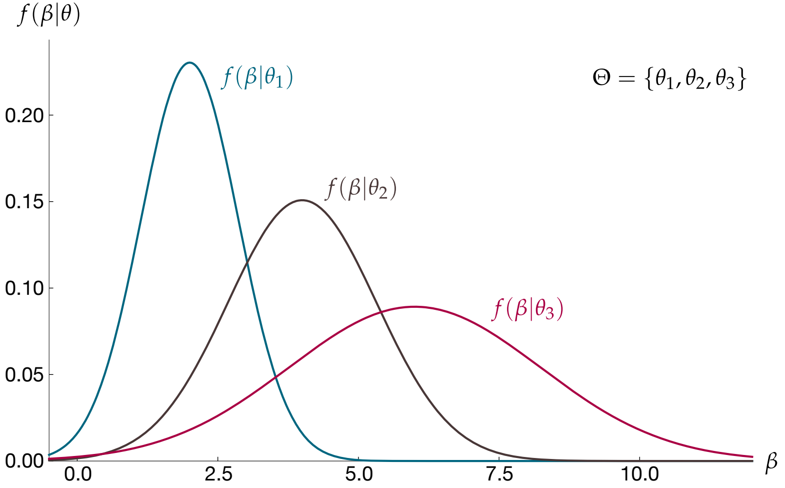
$$\mathbb{E}[u(x_i)] = \int_B u(\bar{y} - D(E;\beta) - C(a_i))f(\beta)d\beta \quad (3)$$

for some $u : \mathbb{R}_+ \to \mathbb{R}$

Modelling ambiguity

- Assume density f of β is unknown
- Estimated by scientific studies, not pinned down yet
- Let $\Theta \subseteq \mathbb{R}$ be the set of all relevant scientific studies
- Denote by $f(\cdot|\theta)$ the density estimated by $\theta \in \Theta$

Modelling ambiguity



Beliefs

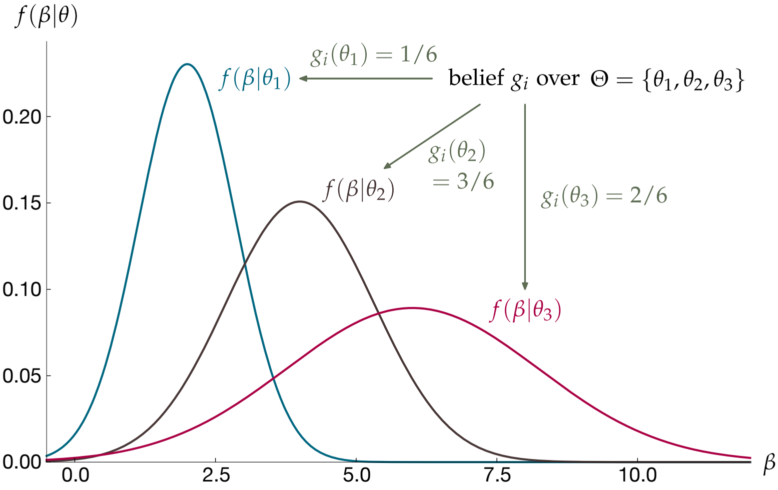
Modelling beliefs

- No a priori information available about the relative credibility of each of the possible densities
- Prior $g_i \in \Delta(\Theta)$ defined over the set of densities
- Specific to each player, subjectively chosen

Heterogeneity in beliefs

- Due not to asymmetric information, but rather to psychological biases
- Suggested by recent experimental evidence (DellaVigna, 2009; Hommes, 2012)
- Belief profile $\{g_i\}_{i=1}^n$ is common knowledge, as in the case of climate change

Illustration of belief



Information structure

Public signal

- About which of the proposed densities correctly captures the inherent risk of β
- Say $f(\cdot|\theta_*)$ is the true risk, where $\theta_* \in \Theta$ is unknown
- Signal $\mu_* \in \Theta$ available upon scientific discoveries:

$$\mu_* = \theta_* + \eta \quad \text{where} \quad \eta \sim N(0, \sigma_*^2) \tag{4}$$

- $\sigma_*^2 \geq 0$ represents ambiguity remaining in science

Updating beliefs

- Once μ_* observed, the posterior $g_i(\cdot|\mu_*)$ is given by:

$$g_i(\theta|\mu_*) \propto L(\mu_*|\theta) g_i(\theta), \tag{5}$$

where $L(\mu_*|\theta)$ is the likelihood of μ_* when $\theta_* = \theta$

Decision making

Decision utility

- Smooth ambiguity model of Klibanoff et al. (2005)
- Players behave so as to maximize

$$V_i := \int_{\Theta} \phi(\mathbb{E}[u_i|\theta])g_i(\theta)d\theta, \tag{6}$$

  where

$$\mathbb{E}[u_i|\theta] := \int_B u(x_i)f(\beta|\theta)d\beta \tag{7}$$

- Uncertainty preference captured by u and ϕ:
  - concavity of u implies risk aversion
  - concavity of ϕ implies ambiguity aversion
- Assume u and ϕ are both concave

Equilibrium and welfare

Equilibrium

- $a := (a_i)_{i=1}^n$ is eqm if a_i maximizes $V_i(a_i, a_{-i})$ for all i
- Belief g_i is replaced by $g_i(\cdot|\mu_*)$ once μ_* observed
- Denote by $\tilde{a} := (\tilde{a}_i)_{i=1}^n$ the eqm corresponding to μ_*

Welfare (as opposed to decision utility)

- Evaluated at the true risk: $W_i^c(a) := \phi(\mathbb{E}[u_i|\theta_*])$
- Since θ_* is unknown, we instead use

$$W_i(a) := \mathbb{E}[W_i^c(a)|\mu_*] = \int_\Theta \phi(\mathbb{E}[u_i|\theta])g_*(\theta), \tag{8}$$

  where g_* is the density of θ_* conditional on μ_*
- Note g_* can be seen as the rational belief
- This pins down the efficient level of A_* and $a_* := A_*/n$

Characterizing equilibrium

First-order condition

- At eqm

$$C'(a_i) = \int_B D'(E;\beta) f_i(\beta) d\beta \quad \forall i, \tag{9}$$

  where

$$f_i(\beta) := \int_\Theta \hat{f}_i(\beta|\theta) \hat{g}_i(\theta) d\theta, \tag{10}$$

$$\hat{f}_i(\beta|\theta) \propto u'(x_i) f(\beta|\theta), \tag{11}$$

$$\hat{g}_i(\theta) \propto \phi(\mathbb{E}[u(x_i)|\theta])\mathbb{E}[u'(x_i)|\theta] g_i(\theta) \tag{12}$$

- MC and 'distorted' MB equalized:
  - $\tilde{f}_i(\beta|\theta)$ is preference-adjusted risk $\leftarrow$ risk pref.
  - $\tilde{g}_i(\theta)$ is preference-adjusted belief $\leftarrow$ risk/amb pref.
- Beliefs and preference both play important roles in MB

Role of beliefs

Well-ordered risks

- Assume the risks $\{f(\cdot|\theta)\}_{\theta\in\Theta}$ are well ordered in the sense of strict monotone likelihood ratio:

$$f(\beta'|\theta')f(\beta|\theta) > f(\beta'|\theta)f(\beta|\theta') \quad \forall\beta' > \beta,\ \forall\theta' > \theta \tag{13}$$

- Then $\theta' > \theta$ implies θ' is more 'pessimistic' than θ
- Examples: normal $N(\theta, \sigma_u^2)$, chi-squared $\chi^2(k, \theta)$

Proposition 1

- For two players i and $j \neq i$, if

$$g_i(\theta')g_j(\theta) > g_i(\theta)g_j(\theta') \quad \forall\theta' > \theta, \tag{14}$$

  then player i abates more than player j at eqm
- Pessimistic beliefs translated into larger abatement

Inefficiency

Due to externality

- Inefficiency arises even under the rational belief ($g_i = g_*$), a consequence of externality
- Even more inefficient if risk is underestimated, i.e.,

$$g_*(\theta')g_i(\theta) > g_*(\theta)g_i(\theta') \quad \forall \theta' > \theta \tag{15}$$

- Rationalization of beliefs ($g_i \to g_*$) is Pareto-improving

Due to heterogeneity

- Prop. 1 suggests heterogeneous beliefs lead to uncoordinated actions
- Inefficiency then follows from convexity of cost function and Jensen's inequality
- Belief convergence ($d(g_i, g_j) \to 0$) improves efficiency

Role of preference

Propositions 4 and 5

- In the presence of ambiguity:
  - risk- and ambiguity-averse players have an extra incentive to abate
  - the more ambiguity averse, the larger abatement
- Kind of precautionary behavior

Potentially negative value of information

- Additional information reduces the existing ambiguity, which counteracts the precautionary incentive
- If this side effect is large enough, players might be all worse off by new information
- We clarify when and in what condition such a paradoxical consequence follows

Value of information: specifications

Basic game

- Specify

$$u(x) := -\frac{1}{\alpha}e^{-\alpha x}, \quad \phi(u) := -\frac{1}{1+\xi}(-u)^{1+\xi}, \tag{16}$$

  where α, ξ are indices of risk and ambiguity aversion

- $D(E;\beta) := \beta\delta E$ and $C(a_i) := (\nu/2)a_i^2$

Uncertainty

- Assume risks/beliefs are well represented by normal:
  - $f(\cdot|\theta) \sim N(\theta, \sigma_u^2)$ with $\sigma_u^2 > 0$
  - $g_i \sim N(\mu_i, \sigma_i^2)$ with $\sigma_i^2 > 0$
- $\mu_i \in \Theta$ is the point estimate of θ_* by player i
- $1/\sigma_i^2$ measures player i's confidence

Equilibrium of specified model

Closed-form solution

- Eqm abatement is

$$a_i = \rho\mu_i + \rho\delta E\gamma_i, \tag{17}$$

where $\gamma_i := \alpha[\sigma_u^2 + (1+\xi)\sigma_i^2]$ and $\rho := \delta/\nu$

- γ_i summarizes uncertainty and preference
- Pessimistic belief (larger μ_i) implies larger abatement
- Greater uncertainty (larger γ_i) implies larger abatement

Inefficiency

- Assume the risk is underestimated in the sense that

$$\mu_i < \mu_*, \quad \sigma_i^2 < n\sigma_*^2 \quad \forall i \tag{18}$$

- This ensures $A := \sum_i a_i < A_*$

Impact of new information

Reshaping players' beliefs

- Recall the public signal is $\mu_* \sim N(\theta_*, \sigma_*^2)$
- Posterior is hence given by $N(\tilde{\mu}_i, \tilde{\sigma}_i^2)$, where

$$\tilde{\mu}_i = \frac{\sigma_*^2}{\sigma_i^2 + \sigma_*^2}\mu_i + \frac{\sigma_i^2}{\sigma_i^2 + \sigma_*^2}\mu_*, \quad \tilde{\sigma}_i^2 = \frac{\sigma_*^2}{\sigma_i^2 + \sigma_*^2}\sigma_i^2 \tag{19}$$

- Three distinct effects observed:
  - rationalization effect: $|\tilde{\mu}_i - \mu_*| < |\mu_i - \mu_*|$
  - convergence effect: $|\tilde{\mu}_i - \tilde{\mu}_j| \to 0$ as $\sigma_*^2 \to 0$
  - confidence (less ambiguity) effect: $\tilde{\sigma}_i^2 < \min\{\sigma_i^2, \sigma_*^2\}$
- One effect dominates the other, depending on priors and preference

When good news turns into bad news

Proposition 6

- A condition for the confidence effect to dominate
- For each (α, ξ), there is $(\Delta\mu, \Delta\sigma^2) \in \mathbb{R}^2_{++}$ such that
  1. if $\sum_i |\mu_* - \mu_i| < \Delta\mu$, then $\tilde{A} < A$
  2. if furthermore $\sum_i |\sigma^2_* - \sigma^2_i| < \Delta\sigma^2$, then $\tilde{W}_i < W_i \; \forall i$
- $(\Delta\mu, \Delta\sigma^2)$ is increasing in (α, ξ)

Policy implications

- Routinely publishing assessment reports with minor updates might do more harm than good
- Even if the risk is underestimated by players
- Should instead be published only when significantly novel findings are available

Information noise

Modified information structure

- Assume information noise can be credibly added
- Players receive a noisy signal μ_*^ε such that

$$\mu_*^\varepsilon = \mu_* + \varepsilon, \quad \varepsilon \sim N(0, \sigma_\varepsilon^2) \tag{20}$$

- Posterior is then given by $N(\tilde{\mu}_i, \tilde{\sigma}_i^2)$, where

$$\tilde{\mu}_i = \frac{\sigma_*^2 + \sigma_\varepsilon^2}{\sigma_i^2 + \sigma_*^2 + \sigma_\varepsilon^2}\mu_i + \frac{\sigma_i^2}{\sigma_i^2 + \sigma_*^2 + \sigma_\varepsilon^2}\mu_*, \tag{21}$$

$$\tilde{\sigma}_i^2 = \frac{\sigma_*^2 + \sigma_\varepsilon^2}{\sigma_i^2 + \sigma_*^2 + \sigma_\varepsilon^2}\sigma_i^2 \tag{22}$$

- Noise affects the rationalization and convergence effects as well as the confidence effect

Pareto-improving ambiguity

Issue of interest

- Preceding analysis is nested in this modified model:
  - $\sigma_\varepsilon^2 = 0$ corresponds to direct-publication case
  - $\sigma_\varepsilon^2 \to \infty$ corresponds to no-information case
- Of interest is if both cases can be Pareto-dominated by some positive yet finite noise $\sigma_\varepsilon^2 \in (0, \infty)$

Definition

- We say that Pareto-improving ambiguity is possible if there exists $\sigma_\varepsilon^2 \in (0, \infty)$ such that

$$\tilde{W}_i(\sigma_\varepsilon^2) > \tilde{W}_i(\sigma_\varepsilon^2)\Big|_{\sigma_\varepsilon^2=0} > \lim_{\sigma_\varepsilon^2 \to \infty} \tilde{W}_i(\sigma_\varepsilon^2) \quad \forall i \tag{23}$$

- Value of information itself is positive (2nd inequality)
- Even better if some noise is added (1st inequality)

Structure of heterogeneity matters

Partial heterogeneity

- If there is no heterogeneity in $\{\sigma_i^2\}_{i=1}^n$, then Pareto-improving ambiguity is impossible
- If players are equally confident about their beliefs, information would have a 'uniform' impact
- Relation between σ_ε^2 and A (thus W_i) is then monotonic

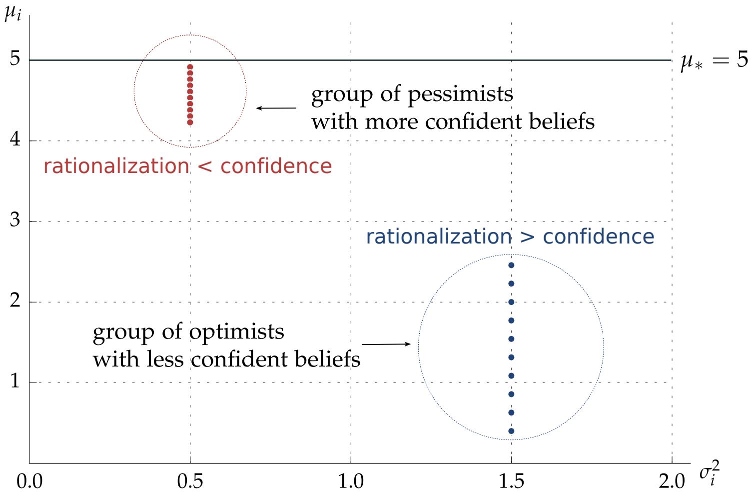
Full heterogeneity

- Non-monotonic relationship is possible if and only if

$$\frac{\mu_* - n^{-1}\sum \mu_i}{n^{-1}\sum \sigma_i^2} > \frac{1}{n}\sum_i \frac{\mu_* - \mu_i}{\sigma_i^2} \tag{24}$$

- Heterogeneity required both in $\{\mu_i\}_{i=1}^n$ and in $\{\sigma_i^2\}_{i=1}^n$
- Confident pessimists and less confident optimists

Illustration of heterogeneous priors



Non-monotonic impact on abatement

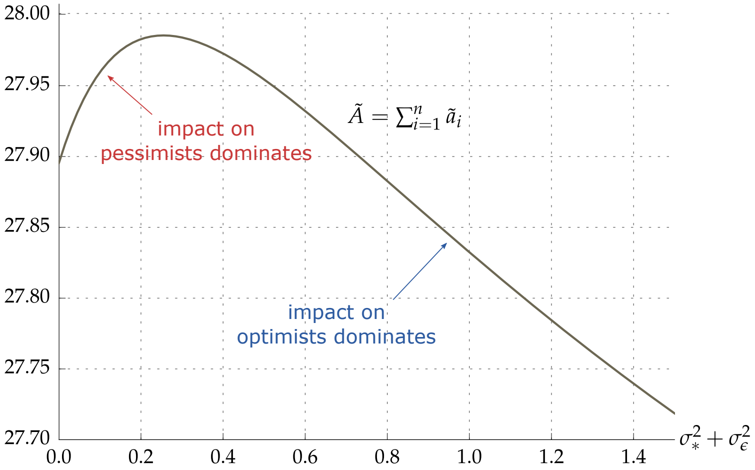
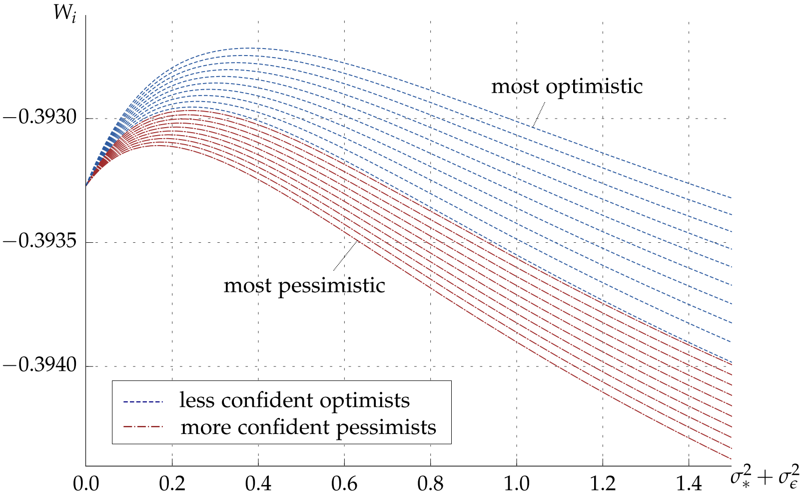


Illustration of Pareto-improvement



Conclusions

Value of information and heterogeneous beliefs

- Important trade-off: the rationalization, convergence, and confidence effects
- Potentially negative value of information even when it better reflects the true risk
- Heterogeneity in beliefs matters, both in terms of its magnitude and of its structure

Directions for future research

- Coalition formation
- Strategic interaction between players and the authority