Dynamic resource management under the risk of regime shifts*

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Abstract

This paper provides a framework through which a dynamic resource management problem with potential regime shifts can be analyzed both in a strategic environment and from a social planner's perspective. Based on a fairly general model, a condition for a precautionary policy is discussed. By applying the framework to a common-property resource problem with a linear production technology, we illustrate how the qualitative as well as quantitative nature of equilibrium is altered due to the possibility of regime shifts. In particular, when the risk is endogenously affected by the players' behavior, potential regime shifts can facilitate the precautionary management of resources as long as the resource stock is in good shape. As the stock of resource becomes scarce, however, the precautionary effect vanishes and more aggressive resource exploitation emerges. The impacts of irreversibility on the equilibrium behavior are highlighted. It is also shown that there can exist a resource-depletion trap in which a regime shift, once it happens, triggers a continuous decline of resource stock no matter which regime materializes in the subsequent periods.

Keywords: regime shifts; uncertainty; irreversibility; optimal management; dynamic games

JEL classification: C61; C73; Q20; Q56

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‡Final version. Some typos are corrected.

1 Introduction

This paper studies a dynamic problem in which a system characterizing the economy shifts from one regime to another at unpredictable points in time. A typical situation described by such a problem is the joint exploitation of common-property resources such as lakes, forests, marine fish populations, and at a larger scale, the global climate system. These resources have complex dynamic systems that are known to undergo sudden and drastic changes in terms of their underlying regimes (Scheffer et al., 2001; Scheffer and Carpenter, 2003; Folke et al., 2004). Shallow lakes, for instance, tend to at some point suddenly lose transparency and vegetation due to the heavy use of fertilizers on the surrounding land and increased inflow of waste water from human settlements and industries (Scheffer et al., 1993). A potential collapse of the West Antarctic Ice sheet or a change in the global ocean circulation is expected to happen quite abruptly and is anticipated to be caused by anthropogenic climate change (Oppenheimer, 1998; Broecker, 1997). Aside from common-property resource problems, dynamic systems with potential regime shifts can also be found in financial markets (Cass and Shell, 1983; Guo et al., 2005).

The existing literature that studies potential regime shifts in dynamic systems has mainly focused on optimal management of the systems. Cropper (1976) considers a regime shift in the form of a catastrophic plunge of utility level triggered by pollution. Reed (1988) determines the socially optimal harvesting policy for a fishery subject to random catastrophic collapse. In a more general setting, Clarke and Reed (1994) considers the impact of a stock-dependent risk of catastrophic environmental collapse on the optimal management of resources, which was later extended by Tsur and Zemel (1996). In these studies, regime shits are commonly formulated as a catastrophic damage or the collapse of resource stock. In a more recent paper, Polasky et al. (2011) considers changed system dynamics as a general type of regime shift and shows that the optimal management of resources with potential regime shifts can be a precautionary measure. A similar kind of precautionary optimal policy was also found by de Zeeuw and Zemel (2012) in the context of pollution control. In this strand of literature, however, strategic aspects inherent in the management of common-property resources are not fully taken into account. Moreover, the analysis is commonly focused upon steady states, especially when the risk of a regime shift is endogenously affected.

There exists vast literature on non-cooperative dynamic games. A seminal paper by Levhari and Mirman (1980), for instance, examines the dynamic and steady-state properties of a fish population that results from strategic interaction among players. Several issues of importance such as the existence, multiplicity, and inefficiency of equilibria have already been discussed by Benhabib and Radner (1992), Dockner and Sorger (1996), and Sorger (1998), among others. Only a few papers, however, incorporate the risk or uncertainty surrounding the joint exploitation of productive resources. Recently, Antoniadou et al. (2013) introduced a simple random shock into the growth function of a common-property resource and identified a class of dynamic games that support a linear symmetric Markov-perfect Nash equilibrium in their setting. Their analysis shows that the existence of uncertainty can amplify or mitigate the commons problem, depending on preference and technology in the economy. Yet consideration of uncertain regime shifts is largely absent in the analysis of dynamic games.

The present paper provides a general framework with which a dynamic resource management problem with potential regime shifts can be analyzed in a strategic environment as well as from a social planner's perspective. Section 2 explains the structure of the model and introduces its basic assumptions. Based on a fairly general model, necessary and sufficient conditions for the solution of each player's problem are derived. Accordingly, a symmetric Markov-perfect Nash equilibrium is defined for the model in a general form. We also discuss a general condition for a precautionary policy under the risk of regime shifts.

Section 3 demonstrates how the framework presented in this paper can be used to investigate problems of interest. To this end, we focus on a dynamic common-property resource problem with a linear production technology. This class of games allows a continuous growth of economy, which in turn makes it possible to illustrate the off-steady-state dynamics under the risk of regime shifts. In order to clarify the implications of potential regime shifts, three different cases are considered: one with no regime shift, one with exogenous risk of regime shift, and one with endogenous risk of regime shift. It is shown that when the risk is affected by the level of resource stock, the potential of the regime shifts can facilitate precautionary management of common-property resources even in a strategic environment. This precautionary behavior is reinforced when the regime shift is irreversible and in particular if the consequence is catastrophic. When the remaining resource stock is small, however, the precautionary effect disappears and the equilibrium extraction by players can be even more agressive than in the absence of a regime shift. Equilibrium dynamics are also examined. In particular, it is shown that unlike the models with no regime shift, there can exist a resource-depletion trap in which a regime shift, once it happens, triggers an irreversible accumulation dynamics in the direction of a resource collapse. Section 4 concludes the paper.

2 General Framework

2.1 Regime

Consider an economy with $N \in \mathbb{N}$ identical players sharing a productive resource. Although our framework allows for the case with a single decision maker, we refer to each individual as a 'player' to fix the context. Let θ be a vector of parameters that characterize the current system of economy. We call θ a *regime*. We assume that the utility of each player in general depends both on the flow and stock of a resource. Therefore the utility function of player $n \in \{1, 2, \ldots, N\}$ is given by

$$U(x_n(t), z(t); \theta), \tag{2.1}$$

where $z(t) \in \mathbb{R}_+$ is the stock of resource and $x_n(t) \in \mathbb{R}_+$ is the extraction by player n at period t. Since the players are identical, we suppress the subscript n whenever confusion is unlikely. We denote a time path of stock and extraction by $\{z(t)\}$ and $\{x(t)\}$, respectively. Technology available in the economy is represented by a growth function G and the accumulation process is expressed as

$$\dot{z}(t) = G(z(t), \textstyle\sum_{n=1}^{N} x_n(t); \theta), \tag{2.2}$$

which governs the relationship between the stock and flow of a resource under a given regime. This also provides a channel through which players can strategically interact.

The economy experiences regime shifts, which we model as discontinuous changes in θ at unpredictable points in time. Let Θ be the set of all possible values of θ. A regime shift, say from $\theta_1 \in \Theta$ to $\theta_2 \in \Theta$, might cause a sudden change in the players' taste, a sharp decline in the productivity of a resource, or both. Such a shift is triggered by stochastic events. The risk of regime shifts is thus captured by a hazard rate

$$\lambda(t) = \lambda(z(t); \theta) \geq 0. \tag{2.3}$$

Notice we allow for the possibility that the hazard rate is influenced by the resource stock. One could interpret this as representing the fact that the current level of resource stock directly affects the frequency of shift-triggering stochastic events. Alternatively, (2.3) could mean that while stochastic events themselves are exogenous, the current regime may or may not survive these events, depending on the state of the resource stock at the timing of stochastic shocks.

The simplest way of modeling a regime shift is to assume that the shift occurs once and only once, as in Polasky et al. (2011) and de Zeeuw and Zemel (2012). In reality, however, regime shifts are better modeled as open-ended processes. The climate in the West African Sahel region, for example, has stable dry and wet regimes and has been shifting back and forth between the two (Nicholson, 2000). Another example can be found in the aquatic ecosystem in the North Pacific Ocean, which has experienced multiple regime shifts for the past forty years (Hare and Mantua, 2000). In general, not only the timing, but also the realized state of regime is unknown *ex ante*. Therefore we assume θ is stochastic and obeys a Markov process. To be more precise, θ is a piecewise deterministic Markov process, which has a stationary distribution over Θ, and the conditional distribution $p(\cdot|\theta)$ of the next regime θ' at the timing of a regime shift only depends on the preceding realization of the regime.

An alternative, and perhaps a more natural formulation for the risk of regime shifts is denoted by $\tilde{\lambda}(z;\theta,\theta')$ the hazard rate of regime switching from θ to θ'. In our formulation, the hazard rate $\lambda(z;\theta)$ models the occurrence of any shift from regime θ while the probability distribution $p(\cdot|\theta)$ models where the system is going to end up after the shift. In other words, we assume a separability of the hazard rate so that we may write $\tilde{\lambda}(z;\theta,\theta') = \lambda(z;\theta)p(\theta'|\theta)$. This approach is slightly less general but greatly simplifies the analysis while keeping the essential characteristics of the regime shift intact.

The model is then represented by a list $\langle N, U, G, \lambda, \Theta, p\rangle$. This general formulation nests other approaches in the existing studies. Polasky et al. (2011), for instance, considers a problem where $N = 1$, U is a linear function of x, and G allows for nonlinearity. Similarly, the pollution control problem considered by de Zeeuw and Zemel (2012) is a model where $N = 1$, U is a quadratic function, and G is a linear function. In Polasky et al. (2011), a regime shift is expected in the growth function while de Zeeuw and Zemel (2012) considers a shift in the damage (and thus utility) function. Both studies assume $\Theta = \{\theta_1, \theta_2\}$ and $p(\theta_2|\theta_1) > p(\theta_1|\theta_2) = 0$ so that the process is irreversible. Many of the existing models of dynamic games in a deterministic environment are special cases of our framework where $N \geq 2$ and Θ is a singleton.

2.2 The Players' Problems

We focus on symmetric Markov-perfect Nash equilibria or MPNE. Denote by $Z \subset \mathbb{R}_+$ the set of possible values of resource stock and by $X(z) \subset \mathbb{R}_+$ the set of admissible values of x given a particular level z of resource stock. Let Θ_p be the support of p, and $\phi : Z \times \Theta_p \to X := \cup_{z\in Z} X(z)$ be a stationary Markovian strategy such that $\phi(z;\theta) \in X(z)$ for all $(z,\theta) \in Z \times \Theta_p$. Suppose at period s, the current regime is given by $\theta \in \Theta_p$. Given that other players choose their extraction according to ϕ, a path $\{x(t)\}$ is feasible from an initial stock $z \in Z$ if $x(t) \in X(z(t))$ and $\dot{z}(t) = G(z(t), (N-1)\phi(z(t);\theta) + x(t);\theta)$ for all $t \geq s$ with

$z(s) = z$. The problem for each player is then formulated as

$$V(z;\theta) = \max_{x(t)\in X(z(t))} \mathbb{E}\bigg[\int_s^T e^{-\rho(t-s)} U(x(t), z(t); \theta) dt + e^{-\rho(T-s)} \mathbb{E}\left[V(z(T);\theta')|\theta\right] \bigg| T \geq s\bigg], \tag{2.4}$$

$$\text{s.t.} \quad \dot{z}(t) = G(z(t), (N-1)\phi(z(t);\theta) + x(t); \theta), \tag{2.5}$$

$$z(s) = z > 0 \text{ given}, \tag{2.6}$$

where $\rho > 0$ is the discount rate and $\mathbb{E}\left[V(z;\theta')|\theta\right] = \int_\Theta V(z;\theta')dp(\theta'|\theta)$. The expectations operator outside of the objective function (2.4) is taken in terms of timing T for the regime shifts.

It is worth noting that since the conditional density of timing T of a regime shift is given by $f(T;\theta|T \geq s) = \lambda(z(T);\theta)e^{-\int_s^T \lambda(z(\tau);\theta)d\tau}$, the objective function (2.4) may be rewritten as

$$\int_s^\infty e^{-\rho(t-s)-\int_s^t \lambda(z(\tau);\theta)d\tau}\Big\{U(x(t), z(t);\theta) + \lambda(z(t);\theta)\mathbb{E}[V(z(t);\theta')|\theta]\Big\}dt. \tag{2.7}$$

This means that the problem can be seen essentially as a deterministic problem within each regime. One could even make it look more familiar by introducing another state variable $y(t) := e^{-\int_s^t \lambda(z(\tau);\theta)d\tau}$, and thus $\dot{y}(t) = -\lambda(z(t);\theta)y(t)$ with $y(s) = 1$, with which (2.4) is treated as a standard autonomous problem with two state variables.

2.3 Equilibrium

In this subsection, equilibrium is defined. Let us first state the necessary condition for the solutions of the problem.

Proposition 1. *Consider a game $\langle N, U, G, \lambda, \Theta, p\rangle$. Suppose that the value function $V : Z \times \Theta_p \to \mathbb{R}$ in (2.4) is well defined and is continuously differentiable in z. Also suppose that the optimal policy of players are characterized by a Markovian strategy $\phi : Z \times \Theta_p \to X$. Let $V' := \partial V/\partial z$. Then V solves*

$$\rho V(z;\theta) = U(\phi(z;\theta), z;\theta) + V'(z;\theta)G(z, N\phi(z;\theta);\theta) + \lambda(z;\theta)\left\{\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta)\right\} \tag{2.8}$$

and ϕ satisfies

$$\phi(z;\theta) \in \operatorname*{argmax}_{x\in X(z)} \Big\{U(x, z;\theta) + V'(z;\theta)G(z, (N-1)\phi(z;\theta) + x;\theta)\Big\} \tag{2.9}$$

for each $(z, \theta) \in Z \times \Theta_p$.

Proof. See Appendix A.1. □

The Hamilton-Jacobi-Bellman (HJB) equation (2.8) might look a bit complicated. To interpret it, notice that (2.8) may be written as

$$\tilde{\rho}(z;\theta)V(z;\theta) = \max_{x\in X(z)} \Big\{U(x,z;\theta) + V'(z;\theta)G(z,(N-1)\phi(z;\theta)+x;\theta)\Big\}, \quad (2.10)$$

where

$$\tilde{\rho}(z;\theta) := \rho + \frac{V(z;\theta) - \mathbb{E}[V(z;\theta')|\theta]}{V(z;\theta)}\lambda(z;\theta). \quad (2.11)$$

Comparing (2.10) with the standard HJB equation, we see that the risk of regime shifts in effect changes the discount rate from ρ to $\tilde{\rho}$. When there is no risk of regime shift, ρ and $\tilde{\rho}$ coincide because in that case $\mathbb{E}[V(z,\theta')|\theta] = V(z;\theta)$ in (2.11). If λ is independent of z and if $\mathbb{E}[V(z;\theta')|\theta] = 0$, which should be the case when the next regime is the 'doomsday,' then $\tilde{\rho} = \rho + \lambda$. This means that the effective discount rate is raised exactly to the extent of the hazard rate, a well-known result from Yaari (1965). In such a case, the HJB equation can be solved just like in the case of no regime shift.

In general, however, how much players effectively discount the future value of a resource stock depends on what kinds of regimes are expected to materialize in the future. As is clear from (2.11), the effective discount rate becomes higher than the original one if and only if $\mathbb{E}[V(z;\theta')|\theta] < V(z;\theta)$. Hence uncertain shifts of regimes do not necessarily imply a higher value of effective discount rate. When players are expected to be better off through a regime shift, for instance, the risk of such a regime shift decreases the effective discount rate. Moreover, the better the coming regimes are, the lower the effective discount rate will be.

Since we assume that λ can depend on z, solving the HJB equation involves extra work. In the standard model of no regime shift, a wide class of differential games have a linear equilibrium in which V and ϕ are both linear in z. When, for example, U and G are both homogenous of degree one, a brief inspection reveals that (2.8) can be satisfied by a linear equilibrium as long as λ is constant. In the case of an endogenous risk of regime shifts, (2.11) indicates that $\tilde{\rho}$ may depend on z in a complicated way. Then even the combination of homogenous functions does not guarantee the existence of a linear equilibrium. This already suggests that the nature of equilibria under the risk of regime shifts will be quite different from the one in the absence of such a risk.

For the sake of completeness, let us state a sufficient condition for the solution, which we can use to check whether a candidate equilibrium strategy in fact solves the original problem.

Proposition 2. *Consider a game characterized by $\langle N, U, G, \lambda, \Theta, p\rangle$. Suppose a continuously differentiable function $V : Z \times \Theta_p \to \mathbb{R}$ and stationary Markovian strategy*

$\phi : Z \times \Theta_p \to X$ satisfy (2.8) and (2.9). If

$$\lim_{T\to\infty} V(\hat{z}(T);\theta)e^{-\int_s^T(\rho+\lambda(\hat{z}(\tau);\theta))d\tau} \geq \lim_{T\to\infty} V(z(T);\theta)e^{-\int_s^T(\rho+\lambda(z(\tau);\theta))d\tau} = 0 \tag{2.12}$$

for each $\theta \in \Theta_p$ and for any feasible stock path $\{\hat{z}(t)\}$ from z, then ϕ solves the problem (2.4) and the corresponding value function is given by V.

Proof. See Appendix A.2. □

With the propositions above in mind, we define equilibrium as follows.

Definition 1. A *symmetric Markov-perfect Nash equilibrium* of game $\langle N, U, G, \lambda, \Theta, p\rangle$ consists of continuously differentiable value function $V : Z \times \Theta_p \to \mathbb{R}$ and stationary Markovian strategy $\phi : Z \times \Theta_p \to X$ such that (2.8), (2.9), and (2.12) are satisfied.

2.4 Condition for a Precautionary Policy

Given the framework presented above, one of the questions of utmost interest is whether some form of precaution emerges at equilibrium as a result of potential regime shifts. In the case with $N = 1$, we can derive a condition for precautionary policy as long as the model has a steady state at equilibrium. To examine this, suppose ϕ and V constitute an equilibrium and V is twice differentiable. If ϕ is an interior solution of (2.9), the first-order condition implies

$$\frac{\partial U(\phi(z;\theta), z;\theta)}{\partial x} = -V'(z;\theta)\frac{\partial G(z, N\phi(z;\theta);\theta)}{\partial x}. \tag{2.13}$$

Differentiating the HJB equation (2.8) and then eliminating V' by (2.13) yields

$$\begin{aligned} &-\left[\rho - \frac{\partial G(z, N\phi(z;\theta);\theta)}{\partial z}\right]\frac{\partial U(\phi(z;\theta), z;\theta)/\partial x}{\partial G(z, N\phi(z;\theta);\theta)/\partial x} \\ &\quad = (N-1)\frac{\partial U(\phi(z;\theta), z;\theta)}{\partial x}\phi'(z;\theta) + V''(z;\theta)G(z, N\phi(z;\theta);\theta) \\ &\qquad + \frac{\partial U(\phi(z;\theta), z;\theta)}{\partial z} + \frac{d}{dz}\left\{\lambda(z;\theta)\left(\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta)\right)\right\}, \end{aligned} \tag{2.14}$$

where $\phi' := \partial\phi/\partial z$ and $V'' := \partial^2 V/\partial z^2$.

If $N = 1$, the first term in the right-hand side of (2.14) vanishes and at a steady state, so does the second term. Let us assume a steady state exists where $z_{ss}(\theta)$ is the steady-state level of the resource stock. We define a function $g : Z \times \Theta \to X$ implicitly by

$$G(z, g(z;\theta);\theta) = 0 \tag{2.15}$$

so that we may write $\phi(z_{ss}(\theta);\theta) = g(z_{ss}(\theta);\theta)$. From (2.14) we have

$$\Xi(z_{ss}(\theta);\theta) = \frac{d}{dz}\left\{\lambda(z;\theta)\left(\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta)\right)\right\}\Big|_{z=z_{ss}(\theta)}, \tag{2.16}$$

where

$$\Xi(z;\theta) := -\left[\rho - \frac{\partial G(z,g(z;\theta);\theta)}{\partial z}\right]\frac{\partial U(g(z;\theta),z;\theta)/\partial x}{\partial G(z,g(z;\theta);\theta)/\partial x} - \frac{\partial U(g(z;\theta),z;\theta)}{\partial z}. \tag{2.17}$$

First, we observe that the steady-state level of the resource stock in the absence of regime shift denoted by $z^*_{ss}(\theta)$ is determined by $\Xi(z^*_{ss}(\theta);\theta) = 0$. Hence, if $\Xi(\cdot;\theta)$ is an increasing function of z at least in a neighborhood of $z^*_{ss}(\theta)$, it then follows that $z_{ss}(\theta) > z^*_{ss}(\theta)$ if and only if

$$\frac{d}{dz}\left\{\lambda(z;\theta)\left(\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta)\right)\right\}\Big|_{z=z_{ss}(\theta)} \tag{2.18}$$

is strictly positive. Notice that $z_{ss}(\theta) > z^*_{ss}(\theta)$ implies a precaution in the sense that the greater amount of resource stock should be saved at a steady state. Hence, the sign of (2.18) determines whether a precautionary policy of this kind emerges or not. This makes sense if we notice that the expression $\lambda(z;\theta)\left(\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta)\right)$ captures the expected welfare change from a regime shift. Suppose, for instance, a welfare loss is expected upon a regime shift so that the expression is negative. Then the positive sign of (2.18) means that the expected welfare loss can be reduced by marginally saving resource stock for the future, a quite intuitive condition for a precautionary policy.

This general condition for the precautionary policy applies to the wide class of models where $N = 1$ and $\Xi(\cdot,\theta)$ is increasing. Moreover, if a regime shift is assumed to happen once and only once, the sign of (2.18) can in principle be determined by solving the problem backwardly. In fact, many of the existing models of regime shift fall into this category[1]. The models with strategic interaction among players, however, are obviously not covered. Also, restricting attention to steady states might fail to capture the implications of a regime shift in other important aspects of the economy such as economic growth rate or equilibrium dynamics in general. In particular, the players' reactions to a changing level of resource stock are not taken into account. This motivates the analysis in the next section, where we turn to a specific dynamic game that does not necessarily have a non-trivial steady state at equilibrium.

[1]See Appendix C for more details.

3 A Common-property Resource Problem

In this section, we demonstrate how the framework presented above can be used to investigate problems of interest and how a different kind of equilibrium emerges as a result of potential regime shifts. To this end, we restrict ourselves to a particular class of games.

3.1 Specifications and Benchmark Results

The class of games we investigate here is a subclass of those analyzed by Sorger (2005), who considers a common-property resource with a constant natural growth rate $R > 0$. The growth function is accordingly given by

$$G(z, \textstyle\sum_{n=1}^{N} x_n; \theta) := Rz - \textstyle\sum_{n=1}^{N} x_n. \tag{3.1}$$

Let $Z = \mathbb{R}_+$ and $X(z) = \mathbb{R}_+$ for $z > 0$ and $X(z) = \{0\}$ for $z = 0$. The players are assumed to derive utility both from the flow of extracted resources and from the existing stock itself. We denote the constant marginal cost of extraction by $\kappa > 0$, which is measured in units of utility. To be more specific, instantaneous utility function is defined by

$$U(x, z; \theta) := x^{\alpha}(\nu z)^{1-\alpha} - \kappa x, \quad \alpha \in (0, 1), \tag{3.2}$$

where $\nu > 0$ captures the quality of amenity service associated with the resource stock. Notice that with this specification, U and G are both homogenous of degree one. For later use, we define a function $u(\beta; \theta) := U(\beta, 1; \theta) = \nu^{1-\alpha}\beta^{\alpha} - \kappa\beta$. Also, let v be the inverse function of $u' := \partial u/\partial\beta$, i.e. $v(\gamma; \theta) := \nu\alpha^{\frac{1}{1-\alpha}}(\kappa + \gamma)^{-\frac{1}{1-\alpha}}$. At the moment, we do not specify the hazard rate function λ. For notational convenience, we denote the parameter in the hazard rate function by ω.

It is worth emphasizing that the model considered here has a wider applicability than it might appear. One possible drawback of this model is the linearity of technology G. This assumption is restrictive when it is interpreted as purely representing a regeneration process of renewable natural resource. The stock z, however, can alternatively be regarded as a composite good that includes both the capital stock existing in the economy as a whole and the stock of natural resource shared by the players[2]. Interpreted this way, the linear technology seems to be less restrictive, capturing the process of economic growth in the broad sense of the term, which is endogenously influenced by the consumption of a natural resource. As another interpretation, x can represent the amount of polluting input while z then measures the quality of the environment, such as

[2]We thank an anonymous referee for suggesting this point.

the climate system. With such an interpretation, the constant R can be naturally regarded as an absorption rate of pollutants in the environment.

In this class of games, a regime is represented by $\theta = (R, \rho, \nu, \alpha, \kappa, \omega)$. Let the set of all possible values of θ be defined by $\Theta := \{\theta \in \mathbb{R}^6_{++} \mid \rho > R, \kappa > \underline{\kappa}(\theta)\}$, where

$$\underline{\kappa}(\theta) := \begin{cases} \max\left\{\frac{\alpha}{N-1}\left(\frac{\nu}{\rho-R}\right)^{1-\alpha}\left(\frac{\alpha N-1}{2-\alpha}\right)^{2-\alpha}, \alpha\left(\frac{\nu N}{R}\right)^{1-\alpha}\right\} & \text{if } \alpha N > 1 \\ \alpha\left(\frac{\nu N}{R}\right)^{1-\alpha} & \text{otherwise.} \end{cases} \tag{3.3}$$

The restriction $\rho > R$ ensures the existence and uniqueness of a linear equilibrium in the absence of the risk of regime shifts (Sorger, 2005). Note that we put another restriction $\kappa > \underline{\kappa}(\theta)$ on the parameter space, requiring that the marginal extraction cost not be too small. This is sufficient for a finite extraction strategy to constitute an equilibrium with a positive growth rate. While this condition is not necessary for most of the analysis below, it enables us to obtain clearer results. To simplify the analysis, we assume that the support $\Theta_p \subset \Theta$ of p is finite. This assumption will be used throughout this paper.

Sorger (2005) studied this type of model in the absence of regime shifts, which corresponds to a special case of our model where Θ_p is a singleton. We take his result as a benchmark against which the role of regime shifts can be evaluated. To state the result, it is useful to define a function $H : \mathbb{R}_+ \to \mathbb{R}$ by

$$H(\gamma;\theta) := u(v(\gamma;\theta);\theta) - Nv(\gamma;\theta)\gamma - (\rho - R)\gamma. \tag{3.4}$$

Appendix B.1 shows that $H(\gamma;\theta) = 0$ has the unique solution and the solution must be strictly positive. Denote the unique positive solution by $\gamma^*(\theta)$ and put $\beta^*(\theta) := v(\gamma^*(\theta);\theta)$. The following proposition characterizes a symmetric MPNE for the benchmark case.

Proposition 3. *Consider a game $\langle N, U, G, \lambda, \Theta, p\rangle$ specified above where $\Theta_p = \{\theta\}$ for some $\theta \in \Theta$. There exists a symmetric MPNE such that $V(z;\theta) = \gamma^*(\theta)z$ and $\phi(z;\theta) = \beta^*(\theta)z$. There are no other symmetric linear MPNE. Moreover, $\gamma^*(\theta)$ is increasing with respect to R and ν, and decreasing with respect to ρ and N. On the other hand, $\beta^*(\theta)$ is decreasing with respect to R and ν, and increasing with respect to ρ and N.*

Proof. See Appendix A.3 □

Note that the constant $\gamma^*(\theta)$ can be interpreted as a welfare index of players at equilibrium. The result makes sense since it states that players are better off when the resource is more productive or when the amenity service from the resource stock is of high quality. On the other hand, the equilibrium level of welfare declines if the players become more impatient or the commons is more crowded. Since $\gamma^*(\theta) = V'(z;\theta)$, the welfare index is in fact the shadow value of z, which explains why the equilibrium extraction rate $\beta^*(\theta)$ moves exactly in the opposite direction. When the shadow value of a resource stock becomes larger, the extraction rate should be smaller and *vice versa*.

As expected, the symmetric MPNE described in the proposition above is inefficient as long as $N > 1$. To see this, consider the case in which all players employ the same linear strategy $x(t) = \beta z(t)$ for some $\beta > 0$. Then $z(t) = ze^{(R-N\beta)(t-s)}$ for all $t \geq s$ and the discounted value of total utility is given by

$$\tilde{V}(\beta;z,\theta) := \int_s^\infty e^{-\rho(t-s)} z(t) U(\beta,1;\theta) dt = \frac{u(\beta;\theta)}{\rho - R + N\beta} z. \tag{3.5}$$

Appendix B.2 shows that $\partial\tilde{V}(\beta^*(\theta);z,\theta)/\partial\beta < 0$ for any $(z,\theta) \in Z \times \Theta$ unless $N = 1$. This means that every player can be better off by simultaneously reducing their equilibrium extraction rate. This game hence exemplifies the tragedy of the commons in a classical sense. Players overexploit the resource at the MPNE and end up with a lower-than-possible level of welfare. A question of particular interest then is whether the risk of regime shifts can encourage players to refrain from overexploitation and to behave in a more precautionary manner[3]. And if such a precautionary resource-use is possible, in what condition is it the case?

3.2 Equilibrium under Risk of Regime Shifts

To answer these questions, we now turn to the case where Θ_p consists of multiple regimes. Then the system switches from one regime to another during the course of dynamic resource management. It should be noted that some distinct regimes can be regarded as essentially the same since different combinations of parameter values can support the same equilibrium. In order to make every occurrence of regime shifts substantial for the players, we define *strictly distinct regimes* as follows.

Definition 2. Let $V^*(\cdot;\theta)$ be the value function in the absence of regime shifts under a particular regime $\theta \in \Theta$. Two regimes $\theta_1 \in \Theta$ and $\theta_2 \in \Theta$ are *strictly distinct* from each other if $V^*(z;\theta_1) \neq V^*(z;\theta_2)$ for some $z \in Z$.

[3]Strictly speaking, the relationship between overexploitation due to the tragedy of the commons and precautionary behavior is not easy to pin down. Once the risk of regime shifts is introduced, not only the equilibrium behavior but also the efficient level of extraction is affected. Therefore, just because the equilibrium extraction rate becomes smaller does not necessarily mean the tragedy of the commons is mitigated. Here we simply say that an equilibrium behavior is precautionary if the extraction rate is smaller than in the benchmark case. Its implication to the tragedy of the commons should be interpreted with some caution.

3.2.1 Exogenous Risk

To introduce the risk of regime shifts, consider first the case in which λ is positive, yet $\lambda' := \partial\lambda/\partial z = 0$. In this case, the hazard rate is independent of the existing level of resource stock. The next proposition shows that there exists a linear equilibrium under the risk of regime shifts just as in the benchmark case.

Proposition 4. *Consider a game $\langle N, U, G, \lambda, \Theta, p\rangle$ specified above where Θ_p contains multiple and strictly distinct regimes with $\lambda(z;\theta) > 0$ and $\lambda'(z;\theta) = 0$ for all $(z,\theta) \in Z \times \Theta_p$. There exists a symmetric MPNE such that $V(z;\theta) = \gamma^x(\theta)z$ and $\phi(z;\theta) = \beta^x(\theta)z$, where $\beta^x(\theta) > \beta^*(\theta)$ and $\gamma^x(\theta) < \gamma^*(\theta)$ for at least one $\theta \in \Theta_p$.*

Proof. See Appendix A.4. □

Proposition 4 states that at least under one particular regime, the potential regime shifts always drive players into more aggressive resource exploitation. On this point, it will be instructive if the equilibrium is expressed in a more explicit form. As shown in Appendix A.4, $\gamma^x(\theta)$ and $\beta^x(\theta)$ satisfy $\beta^x(\theta) = v(\gamma^x(\theta);\theta)$ and

$$\gamma^x(\theta) - \mathbb{E}[\gamma^x(\theta')|\theta] = H(\gamma^x(\theta);\theta)/\lambda(\theta), \tag{3.6}$$

where H is strictly decreasing. Since $H(\gamma^*(\theta);\theta) = 0$ and v is strictly decreasing, this implies $\beta^x(\theta) > \beta^*(\theta)$ if and only if $\gamma^*(\theta) > \mathbb{E}[\gamma^x(\theta')\big|\theta]$. Hence, the exogenous risk of regime shifts accelerates resource extractions when the players are expected to be worse off under the coming regimes than in the case of the continuation of the current regime. If there are strictly distinct regimes, there is at least one regime that is unambiguously better than the other regimes. Under such a regime, any shift is undesirable, which provides an incentive for the players to accelerate their extraction before the shift happens.

To illustrate the point, suppose there are only two regimes possible under p so that $\Theta_p = \{\theta_1, \theta_2\}$. Let us say that the economy is currently characterized by regime θ_1 and the regime is expected to shift into θ_2 at an unpredictable point in time. In many cases, such regime shifts in an ecological system due to human intervention, the regimes are expected to shift in a bad direction. Perhaps the growth rate declines ($R_2 < R_1$), the quality of amenity value decreases ($\nu_2 < \nu_1$), or the players become more impatient ($\rho_2 > \rho_1$). Once such a shift happens, the players are likely to be worse off, which is naturally translated into $\mathbb{E}[\gamma^x(\theta')|\theta_1] < \gamma^*(\theta_1)$. As a result, the players accelerate strategic exploitation, anticipating a decline in the shadow value of a resource stock in the future. This argument is formalized by the next corollary.

Corollary 1. *Consider a game $\langle N, U, G, \lambda, \Theta, p\rangle$ specified in Proposition 4 above with $\Theta_p = \{\theta_1, \theta_2\}$. If $R_1 \geq R_2$, $\nu_1 \geq \nu_2$, and $\rho_1 \leq \rho_2$, where at least one inequality is strict, then $\beta^x(\theta_1) > \beta^*(\theta_1)$ and $\beta^x(\theta_2) < \beta^*(\theta_2)$.*

Proof. See Appendix A.5. □

There is good news and bad news. If the next regime is expected to be more preferable than the current one, which is the case when the current regime is θ_2, the extraction rate becomes smaller. This is because the value of the resource stock is expected to rise in the future and therefore the players will try to preserve the resource. When an undesirable regime shift is expected, however, which is more likely in reality, our result provides a pessimistic prediction. It basically predicts that the extraction rate will be even more intensified than in the benchmark case. This means that the players exploit a resource more aggressively if it is expected to become more difficult to achieve sustainable resource management in the future, which is exactly when such reckless behavior should be avoided.

3.2.2 Endogenous Risk

The pessimistic prediction discussed above might be altered when the risk of regime shift is endogenous and if the players correctly recognize that their behavior affects the risk. To investigate this point, we introduce the following assumption on λ[4].

Assumption 1. There exists $\bar{z} \in \mathbb{R}_{++}$ such that $\lambda(z;\theta) > 0$ for $z < \bar{z}$ and $\lambda(z;\theta) = 0$ for $z \geq \bar{z}$. Moreover, $\lambda(z;\theta)$ is twice continuously differentiable on $(0, \bar{z}]$.

For example, λ satisfies this assumption if it is specified as

$$\lambda(z;\theta) := \begin{cases} \omega \ln(\bar{z}/z) & z \leq \bar{z} \\ 0 & z > \bar{z}, \end{cases} \tag{3.7}$$

where $\omega > 0$ represents the sensitivity of the hazard rate to the level of stock remaining. As depicted in Figure 1, this specification suggests that the regime becomes more resilient as the resource stock grows. If the stock level is sufficiently large (i.e. when $z \geq \bar{z}$), the regime is not susceptible to shift-triggering shocks and so $\lambda(z;\theta) = 0$.

For simplicity, we restrict ourselves to the case where there are only two regimes possible. There is no linear equilibrium in this case. In fact, the HJB equation becomes highly nonlinear and analytic solutions are not available. The characterization of the equilibrium is still possible, though, which we will see in the next proposition.

[4] In Appendix D, we discuss how our results below depend on this assumption.

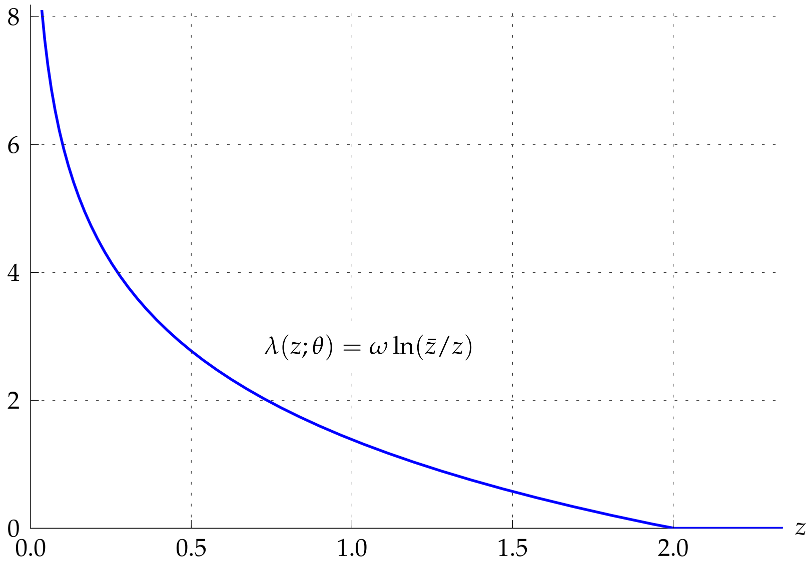

Figure 1: An example of a hazard rate function ($\bar{z} = 2.0, \omega = 2.0$)

Proposition 5. *Consider a game $\langle N, U, G, \lambda, \Theta, p \rangle$ specified above where $\Theta_p = \{\theta_1, \theta_2\}$ and $\lambda(z;\theta)$ satisfies Assumption 1 for each $\theta \in \Theta_p$. Suppose θ_1 and θ_2 are strictly distinct and assume $\gamma^*(\theta_1) > \gamma^*(\theta_2)$ without loss of generality. There exists a symmetric MPNE such that $V(z;\theta) = \gamma^l(z;\theta)z$ and $\phi(z;\theta) = \beta^l(z;\theta)z$, where $\gamma^l(z;\theta) = \gamma^*(\theta)$ and $\beta^l(z;\theta) = \beta^*(\theta)$ for $z \geq \bar{z}$. Moreover, if $\lim_{z \nearrow \bar{z}} \lambda'(z;\theta_1) < 0$, there exists $z_1^\star < \bar{z}$ such that $\beta^l(z;\theta_1) < \beta^*(\theta_1)$ for all $z \in (z_1^\star, \bar{z})$, whereas if $\lim_{z \nearrow \bar{z}} \lambda'(z;\theta_2) < 0$, there exists $z_2^\star < \bar{z}$ such that $\beta^l(z;\theta_2) > \beta^*(\theta_2)$ for all $z \in (z_2^\star, \bar{z})$.*

Proof. See Appendix A.6 □

Although the MPNE described in Proposition 5 might not be unique, it is comparable to the benchmark case in the following sense. As z approaches $\bar{z}$, the hazard rate λ approaches 0, and thereby the problem coincides with the benchmark case. Hence, a natural candidate of equilibrium is the one where the equilibrium extraction rate converges to $\beta^*(\theta)$ as $z \to \bar{z}$, which we focus on in the proposition. In such equilibrium, the value function and policy function both coincide with those in the benchmark case when the resource stock is very large. The equilibrium behavior when the stock level is less than $\bar{z}$, however, is not obvious. When $z < \bar{z}$, the players' actions affect the risk of a regime shift through the associated change in the stock level. Once the risk of a regime shift is altered, the effective discount rate of the players changes, which then provides an incentive for the players to adjust their action. The interaction between the players' actions and the risk of a regime shift is therefore a feedback process.

If $\lambda'(z;\theta) < 0$, this feedback is positive. Suppose, for instance, the current

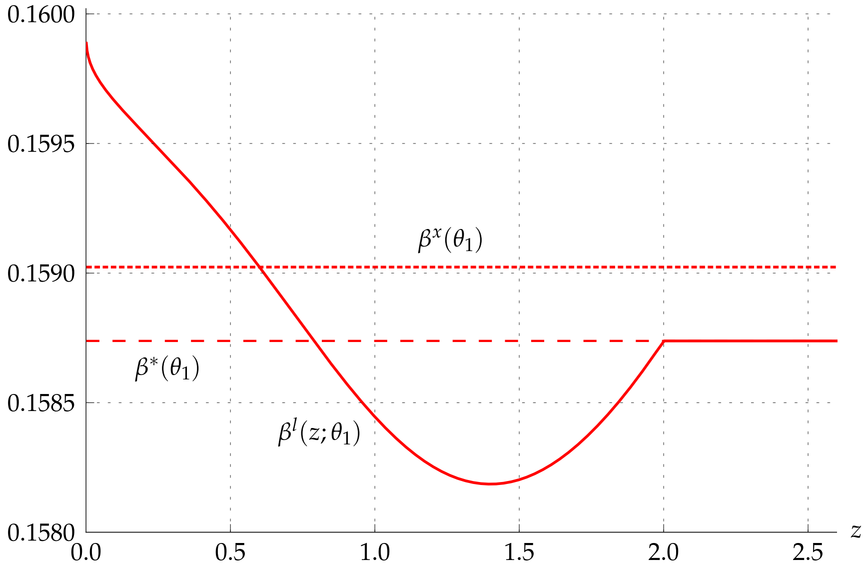

Figure 2: Numerical illustration of equilibrium extraction rate. ($N = 5$, $\bar{z} = 2.0$, $\omega = 2.0$, $\alpha = 1/3$, $\nu = 1.0$, $\kappa = 1.0$, $p(\theta_2|\theta_1) = p(\theta_1|\theta_2) = 0.9$, $\rho = 4.0$, $R_1 = 2.0, R_2 = 1.75$)

economy is characterized by a relatively desirable regime θ_1. When players refrain from the exploitation of a resource, it will decrease the risk of a shift to an undesirable regime θ_2 in the future. This means that the value of resource stock will be more likely to remain high, which in turn implies that players become even more willing to refrain from exploitation. This effect on one hand suggests that equilibrium extraction can be precautionary. The same feedback effect, on the other hand, can go in the opposite direction. When the players accelerate their extraction, it will increase the risk of a regime shift in the future. This means that the value of the resource stock will be more likely to drop in the near future, which in turn provides yet another incentive to accelerate resource extraction.

What is shown by Proposition 5 is that if the remaining resource stock is sufficiently large (i.e. if $z > z_1^\star$), the former feedback effect, the good one, dominates. In this case, we can conclude that the endogenous risk of regime shifts facilitates resource preservation. In fact, the equilibrium extraction rate can be even smaller than in the case of no regime shift despite the fact that the effective discount rate is larger due to the risk of regime shift. Hence, contrary to the case of exogenous regime shifts, the risk of a regime shift does not necessarily imply the acceleration of strategic resource exploitation even if the regime is expected to shift in a bad direction.

Figure 2 provides an illustration of this result. For this numerical example,

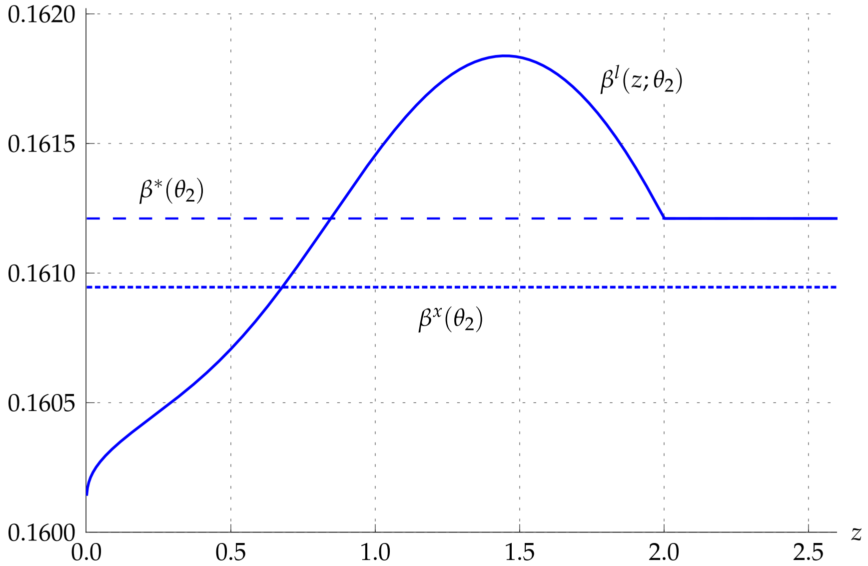

Figure 3: Numerical illustration of equilibrium extraction rate (cont'd).

we specified the hazard rate function as (3.7) and assumed that two regimes are symmetric except for the natural growth rate R of a resource stock. To be more precise, the natural growth rate is assumed to drop from 2.0 to 1.75 once a regime shift happens. The figure shows that in a wide interval of the state space, the equilibrium extraction rate $\beta^l(z;\theta_1)$ remains below the level of the benchmark case as well as of the case with exogenous risk. We thus observe precautionary resource use under the risk of regime shifts. What is also clear from the figure, however, is that as the level of remaining resource stock declines, the extraction rate turns upward at some point and eventually, the precautionary effect disappears. Furthermore, when the remaining stock is sufficiently small, the extraction rate can be even larger than in the case of an exogenous regime shift. This suggests that as the system becomes less resilient, the feedback effect of the second kind, the bad one, gradually kicks in and ultimately outweighs the good feedback effect.

The result is symmetric when the current economy is characterized by a relatively undesirable regime θ_2. Notice that in this case a regime shift is rather attractive. Hence, based on the analogous argument, one would expect that players to intensify the extraction rate under the endogenous risk of a regime shift. The proposition states that this is actually the case when the remaining resource stock is large. As an illustration, Figure 3 depicts the extraction rate under regime θ_2 with the same numerical specification as in Figure 2. If the level of resource stock is quite large, there only exists a slight risk of a regime shift, which is not good when you are currently stuck in a bad regime. Thus the players try to improve the chance of switching regimes by accelerating their extraction activities. As the resource stock declines, on the other hand, the hazard rate increases, implying that the value of the resource stock will be more likely to rise in the future. This provides an incentive for the players to preserve a resource so that they can enjoy a higher level of welfare once a shift happens, which explains why the extraction rate becomes small when z decreases.

3.2.3 Irreversible and Catastrophic Regime Shifts

In the discussion above, we have assumed that even if the system switches from one regime to another, it can still switch back to the original one at some point in the subsequent periods. This assumption is reasonable when the structure underlying the risk of the regime shift is more or less the same in every regime. In some cases of interest, however, the likelihood of a regime shift is highly asymmetric across different regimes and the process can be regarded as rather irreversible. Shallow lakes, for instance, are known to have a pronounced degree of hysteresis in response to nutrient loading (Scheffer et al., 2001). Once a shallow-lake system experiences a regime shift, it then requires substantial efforts to bring the system back to its original regime. Another example of irreversible regime shifts is the one with catastrophic damages. If the magnitude of damage caused by a regime shift is extremely large, as expected in the case of climate change, an occurrence of such a regime shift is so devastating that it might not be possible to recover the system in the foreseeable future.

In order to clarify the implications of irreversibility in regime shifts, we compare two different cases of regime shifts: one with reversible regime shifts and the other with irreversible shifts. To be more formal, consider two games $\langle N, U, G, \lambda, \Theta, p \rangle$ and $\langle N, U, G, \lambda, \Theta, \hat{p} \rangle$, which are identical except for the transition probability distributions p and $\hat{p}$. First suppose $\Theta_p = \Theta_{\hat{p}} = \{\theta_1, \theta_2\}$, i.e. the possible regimes under two games are the same. We assume $p(\theta_2|\theta_1) > 0$ and $p(\theta_1|\theta_2) > 0$ so that regime shifts are reversible under p. Under $\hat{p}$, on the other hand, we assume $\hat{p}(\theta_2|\theta_1) > 0$ and $\hat{p}(\theta_1|\theta_2) = 0$, meaning that a shift from θ_1 to θ_2 is irreversible. By comparing the equilibrium extraction rates in these two games, we can identify the impact of irreversibility itself on precautionary behavior.

Proposition 6. *Consider games $\langle N, U, G, \lambda, \Theta, p \rangle$ and $\langle N, U, G, \lambda, \Theta, \hat{p} \rangle$ specified above where $\Theta_p = \Theta_{\hat{p}} = \{\theta_1, \theta_2\}$ and $\lambda(z;\theta)$ satisfies Assumption 1 with $\lambda'(\bar{z};\theta) < 0$. Suppose θ_1 and θ_2 are strictly distinct with $\gamma^*(\theta_1) > \gamma^*(\theta_2)$. Let $\beta^l_p(z;\theta)$ and $\beta^l_{\hat{p}}(z;\theta)$ be the equilibrium extraction rate described in Proposition 5 under these games, respectively. If $p(\theta_2|\theta_1) = \hat{p}(\theta_2|\theta_1) > 0$, and $p(\theta_1|\theta_2) > \hat{p}(\theta_1|\theta_2) = 0$, then there exists $\hat{z} < \bar{z}$ such that $\beta^l_{\hat{p}}(z;\theta_1) < \beta^l_p(z;\theta_1) < \beta^*(\theta_1)$ for all $z \in (\hat{z}, \bar{z})$.*

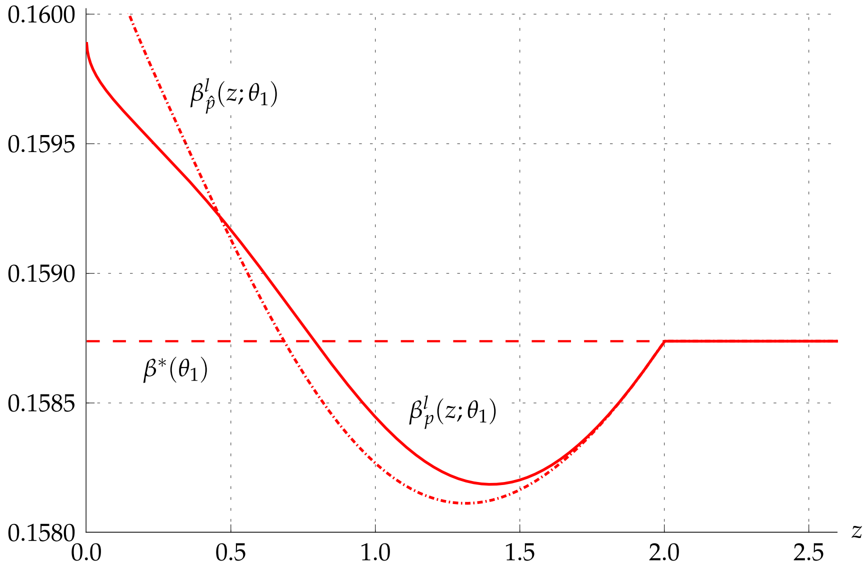

Figure 4: Irreversibility effect on equilibrium extraction rate. Numerical specification is the same as in Figure 2 except for $\hat{p}(\theta_2|\theta_1) = 0$.

Proof. See Appendix A.7. □

Notice first that in Proposition 6, we only focus on the extraction rate in regime θ_1. This is because when a shift from regime θ_1 to regime θ_2 is irreversible, the players' problems under regime θ_2 coincide with the benchmark case. The equilibrium extraction rate in regime θ_2 is simply given by $\beta^l_{\hat{p}}(z;\theta_2) = \beta^*(\theta_2)$ for all z. The impact of irreversibility is hence summarized in the equilibrium extraction rate $\beta^l_{\hat{p}}(z;\theta_1)$ in regime θ_1.

The proposition shows that when an undesirable regime shift is irreversible, the equilibrium extraction rate is smaller than in the case of reversible regime shifts unless the remaining resource stock is small. Hence, we can conclude that the irreversibility of the regime shift reinforces the precautionary effect as long as the resource stock is not in a dire state. To quantify its impact, we continue with the same numerical example as above. The computed equilibrium extraction rate is depicted in Figure 4. As shown in the figure, in a fairly wide interval of the state space, the players extract the resource in a more precautionary manner when the regime shift is irreversible. It is also worth noting that for small z the extraction rate under an irreversible regime shift is larger than in the case of a reversible regime shift. Hence, the irreversibility could backfire if the current level of resource stock is scarce. In either case, however, the impact of the irreversibility itself does not seem large.

The role of irreversibility is a lot more pronounced when the regime shift is catastrophic. To formalize this point, we now compare two cases of irreversible regime shifts: one with a non-catastrophic effect and the other with a catastrophic consequence. Consider games $\langle N, U, G, \lambda, \Theta, \hat{p}\rangle$ and $\langle N, U, G, \lambda, \Theta, \hat{p}^c\rangle$, where $\Theta_{\hat{p}} = \{\theta_1, \theta_2\}$ and $\Theta_{\hat{p}^c} = \{\theta_1, \theta_2^c\}$. Notice that the possible regimes under the two games are not identical. We assume $\hat{p}(\theta_1|\theta_2) = \hat{p}^c(\theta_1|\theta_2^c) = 0$ and $\gamma^*(\theta_2) > \gamma^*(\theta_2^c)$ so that a regime shift is irreversible in both games but the consequences are different. In particular we assume $\gamma^*(\theta_2^c) = 0$ so that the irreversible regime shift under game $\langle N, U, G, \lambda, \Theta, \hat{p}^c\rangle$ is catastrophic.

Proposition 7. *Consider games $\langle N, U, G, \lambda, \Theta, \hat{p}\rangle$ and $\langle N, U, G, \lambda, \Theta, \hat{p}^c\rangle$ specified above where $\Theta_p = \{\theta_1, \theta_2\}$, $\Theta_{\hat{p}} = \{\theta_1, \theta_2^c\}$ and $\lambda(z;\theta)$ satisfies Assumption 1 with $\lim_{z\nearrow\bar{z}} \lambda'(z;\theta) < 0$. Let $\beta^l_{\hat{p}}(z;\theta)$ and $\beta^l_{\hat{p}^c}(z;\theta)$ be the equilibrium extraction rate described in Proposition 5 under these games, respectively. Suppose $\hat{p}(\theta_2|\theta_1) = \hat{p}^c(\theta_2^c|\theta_1) > 0$, and $\hat{p}(\theta_1|\theta_2) = \hat{p}(\theta_1|\theta_2^c) = 0$. If $\gamma^*(\theta_1) > \gamma^*(\theta_2) > \gamma^*(\theta_2^c) = 0$, there exists $\hat{z}^c < \bar{z}$ such that $\beta^l_{\hat{p}^c}(z;\theta_1) < \beta^l_{\hat{p}}(z;\theta_1) < \beta^*(\theta_1)$ for all $z \in (\hat{z}^c, \bar{z})$.*

Proof. See Appendix A.8. □

Proposition 7 simply states that the precautionary effect due to an irreversible regime shift is the largest when the shift triggers a catastrophic consequence. In our model, the catastrophic consequence can be interpreted as a collapse of resource base ($R_2 = 0$) or a complete loss of amenity service from the resource stock ($\nu_2 = 0$). The risk of an irreversible and catastrophic regime shift of this sort facilitates precautionary use of the resource more than any other type of regime shifts. Figure 5 illustrates its quantitative magnitude. For a relatively large z, the equilibrium extraction rate under a catastrophic regime shift is much smaller than in the reversible regime shift and the difference can be more easily appreciated than in Figure 4. It is also quite noticeable that aggressive exploitation when the remaining stock is scarce is greatly intensified as well.

The analysis presented in this section relates to the recent works of Polasky et al. (2011) and de Zeeuw and Zemel (2012). Based on a linear utility model combined with a fairly general growth function, Polasky et al. (2011) considered a social planner's problem with potential regime shifts. They have shown that when the risk of a regime shift is increased by a dwindling resource stock, the optimal management of a productive resource must be precautionary in the sense that the optimal steady-state level of the resource stock is always higher. de Zeeuw and Zemel (2012) investigated a pollution-control problem with an uncertain shift in the damage function and have observed a similar pattern of precaution in the steady state. Our analysis indicates the robustness of their results in the sense that the similar argument holds even in a decentralized and strategic environment. In fact, the mechanism at work is quite similar here. To see this, observe that if an equilibrium exists, it must satisfy the first-order

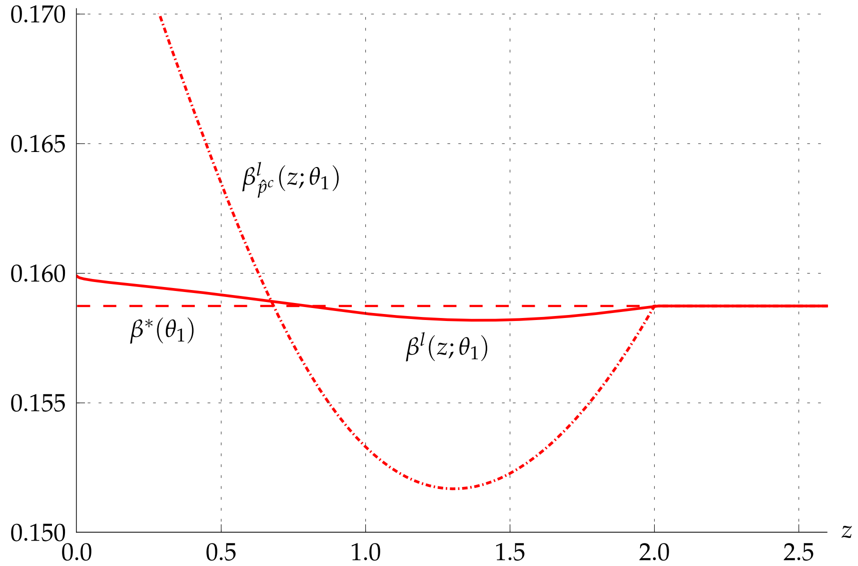

Figure 5: Catastrophe effect on equilibrium extraction rate. A shift to regime θ_2^c is assumed to be irreversible (i.e. $\hat{p}^c(\theta_1|\theta_2^c) = 0$) and catastrophic (i.e. $V(z;\theta_2^c) = 0$). The numerical specification for regime θ_1 is the same as in Figure 2.

condition $u'(\beta(z;\theta);\theta) = V'(z;\theta)$ and the HJB equation, which may be written as

$$\rho V(z;\theta) = \left\{H(u'(\beta(z;\theta);\theta);\theta) + \rho u'(\beta(z;\theta);\theta)\right\} z + \lambda(z;\theta) \left(\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta)\right). \tag{3.8}$$

Differentiating this with respect to z then yields

$$\begin{aligned} &- \left[H'(u'(\beta(z;\theta);\theta);\theta) + \rho\right] u''(\beta(z;\theta);\theta) z \frac{\partial \beta(z;\theta)}{\partial z} \\ &\quad = H(u'(\beta(z;\theta);\theta);\theta) + \frac{d}{dz}\left\{\lambda(z;\theta)\left(\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta)\right)\right\}. \end{aligned} \tag{3.9}$$

Appendix A.6 shows that the expression in front of $\partial\beta/\partial z$ is strictly positive and the first term on the right-hand side vanishes at $z = \bar{z}$. Then what really determines the sign of $\lim_{z \nearrow \bar{z}} \partial\beta/\partial z$ is the second term on the right-hand side of (3.9). If this term is positive at $z = \bar{z}$, the equilibrium extraction rate converges to $\beta^*(\theta)$ from below as $z \to \bar{z}$, implying that the extraction rate is smaller than in the benchmark case, at least in a neighborhood of $\bar{z}$. This condition for precautionary behavior coincides with what we identified for steady-state analysis in Section 2.4.

On the other hand, our analysis also suggests that these precautionarity results under the risk of a regime shift should be more carefully interpreted. We have seen that while common-property resources with potential regime shifts could be better managed than in the absence of a regime shift, this is only the case when the current resource base is in good shape. When the stock of resources is scarce, the precautionary effect vanishes and more agressive resource exploitation emerges. This indicates that the right-hand side of (3.9) becomes rather negative as z becomes smaller.

Recall that our formulation encompasses the case of $N = 1$. Hence, the observed difference is not solely attributed to the consideration of strategic interaction. What drives the difference lies in the fact that while the existing models are mainly focusing on a steady-state analysis, we have investigated the impact of regime shift on a continuously growing economy[5]. As was mentioned in Section 2.4, the steady-state analysis does not fully capture the otherwise complex interplay between the stock-dependent risk of regime shifts and the optimal strategy of decision makers. In particular, the players' reactions to the changing level of a resource stock are not taken into account as long as the attention is restricted to steady states. This can be seen in (2.14), where neither V'' nor ϕ' play a role in determining the steady-state level of the resource stock when $N = 1$. Our analysis therefore suggests the importance of off-steady-state analysis as well as the strategic interaction if the implications of regime shifts are to be fully understood.

3.3 Equilibrium Dynamics

Aside from the precautionary use of resources, the non-linearity of the equilibrium extraction rate produces an interesting consequence in equilibrium dynamics. To highlight the point, first consider the case when the risk of a regime shift is absent or exogenous. Then the equilibrium extraction rate is constant under each regime and the players' strategies can be expressed as $x = \beta z$. Then the process of resource accumulation is given by

$$\dot{z}(t) = Rz(t) - Nx(t) = (R - N\beta)z(t), \tag{3.10}$$

implying that the equilibrium growth rate is constant, at least within the same regime. This means $\dot{z}(t) > 0$ if and only if $R - N\beta > 0$. Since β is a function of θ, the equilibrium dynamics is completely determined by the regime itself, independent of the level of z. In this case, there only exists a trivial steady state with $z = 0$, unless β happens to be equal to R/N.

When the risk of a regime shift is endogenous, on the other hand, the equilibrium extraction rate $\beta^l(z;\theta)$ is a function of the remaining resource stock z.

[5]The analysis of Polasky et al. (2011) has covered the dynamics in general but the linearity of the utility function and the resulting bang-bang control policy in effect invalidate the impact of a regime shift off the steady state. Also, while the model considered by de Zeeuw and Zemel (2012) allows full dynamic characterization of an optimal policy for the case of a constant hazard rate, the implication of a stock-dependent hazard is examined at a steady state.

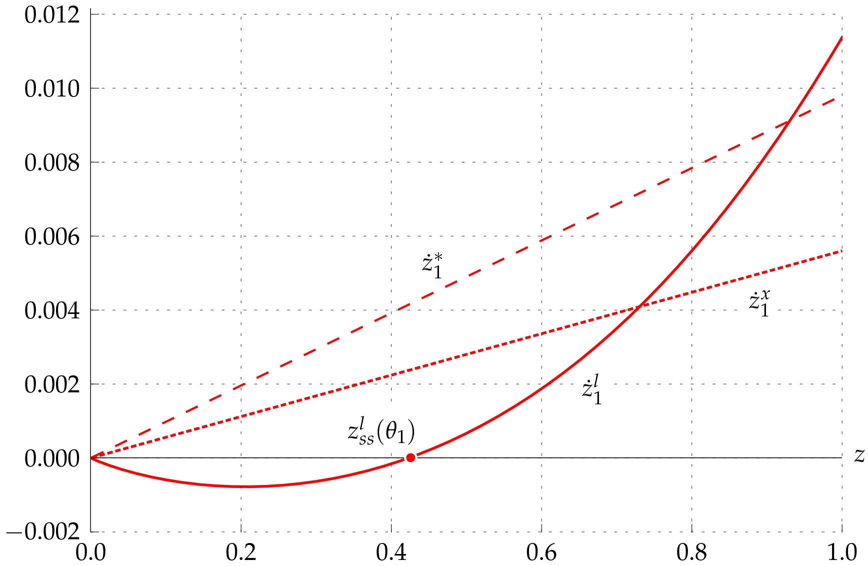

Figure 6: Equilibrium dynamics in regime θ_1 with the same numerical specification as in Figure 2 except for $\kappa = 0.479$.

This implies that there can exist a non-trivial steady state and thereby the equilibrium dynamics depend on the level of the resource stock. In the numerical example depicted in Figure 6, we in fact find a non-trivial and unstable steady state. When the resource stock is larger than the steady-state level $z_{ss}^l(\theta_1)$, the stock continues to grow as long as the current regime persists. The growth rate can be even larger than in the absence of a regime shift when z is sufficiently large. Since the hazard rate is decreasing in z, the risk of a regime shift will then decline over time, providing further basis for the continuous growth of the resource stock. If the stock level is smaller than the steady-state level, however, the logic is completely turned around. The resource stock will gradually diminish, followed by an ever-more frequent occurrence of regime shifts. This starkly contrasts with the case of no or exogenous regime shifts, in which the equilibrium growth rate is always positive.

In this numerical example, the marginal extraction cost κ is set to be small so that the negative growth rate is possible at equilibrium[6]. Since the natural regeneration rate R is assumed to drop when the system switches from θ_1 to θ_2, the dynamic path of z in regime θ_2 is in fact characterized by a continuous decline of the resource stock. As shown in Figure 7, this is independent of

[6]Notice that in this numerical example, the restriction (3.3) is not satisfied. This means that the existence of equilibrium is not ensured in general since the solution to the differential equation (A.34) might not have global support. Nevertheless, this example in fact has the equilibrium described in Proposition 5.

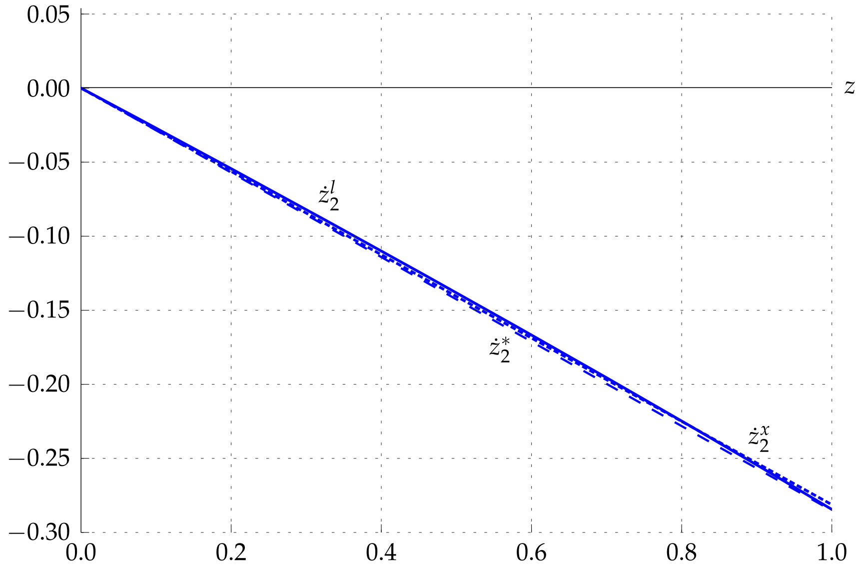

Figure 7: Equilibrium dynamics in regime θ_2 with the same numerical specification as in Figure 6.

whether the risk of a regime shift is absent, exogenous, or endogenous. Hence, once the system switches from θ_1 to θ_2, the resource stock declines thereafter, at least until the next regime shift occurs. It is possible that the next regime shift can trigger a reversal of resource dynamics in the opposite direction. If the risk of a regime shift is exogenous, for instance, the equilibrium growth rate $\dot{z}_1^x$ under regime θ_1 is positive. Therefore the players can always bring themselves back to the process of resource accumulation when the system experiences a regime shift to θ_1. As a consequence, the level of the resource stock oscillates over time.

The situation is different for the case of an endogenous regime shift. What is critical then is the level of resource stock at the time of the regime shift. While the resource stock can oscillate as in the case of an exogenous regime shift, there are 'points of no return' in the stock level, above or below which the equilibrium dynamics are never reversed. If at some period t the remaining resource stock $z(t)$ becomes smaller than the steady-state level $z_{ss}^l(\theta_1)$, then the stock level declines thereafter no matter which regime emerges in the subsequent periods. Therefore the process is irreversible, destined for the collapse of the resource base. This implies on one hand that there exists a resource-depletion trap. Once the system is caught in the trap, it will be impossible to escape from the trap. If on the other hand the resource stock reaches $\bar{z}$ at some period, then one can safely say that the equilibrium resource-use remains sustainable thereafter, since the probability of a regime shift becomes zero. This equilibrium dynamics can also be characterized in terms of initial stock z. When the initial resource stock z is smaller than $z^l_{ss}(\theta_1)$, then the resource management is not sustainable and the resource base diminishes over time. When z is larger than $\bar{z}$, then the resource base never collapses. If z is in between $z^l_{ss}(\theta_1)$ and $\bar{z}$, then the resource management may or may not be sustainable, depending on the realization of regimes in the course of resource exploitation.

It is conjectured that if regime shifts with more than three possible regimes are allowed, each regime may have its own dynamics and steady state. Upon an occurrence of a regime shift, the equilibrium dynamics and the level of steady-state stock are altered accordingly. This implies that when the level of remaining resource stock is 'overtaken' by the steady state under the new regime, the resource stock will thereafter decline. Once the remaining stock becomes smaller than the lowest steady state, the system would never come back to the accumulation path with a positive growth rate. It should be mentioned, however, that the discussion above depends on numerical specification. In general, a non-trivial steady state might not exist or there can be multiple steady states under a particular regime. Hence, the analysis in this section does not always apply. Still, this numerical example demonstrates well the fact that equilibrium dynamics of different kinds can emerge once the risk of a regime shift is taken into account.

4 Conclusions

This paper presented a general framework with which a dynamic problem with potential regime shifts can be analyzed in a decentralized and strategic environment as well as from a social planner's perspective. Based on a fairly general framework, we discussed the necessary and sufficient conditions for the solution of the players' problems and defined symmetric Markov-perfect Nash equilibria for the model of a general form. A general condition for a precautionary policy was briefly discussed. We then demonstrated how the framework presented in this paper can be used to investigate problems of interest.

Using a model of a dynamic common-property resource problem with a linear production technology, we showed that when the risk is endogenous, the potential of regime shifts can facilitate precautionary management of common-property resources as long as the remaining resource stock is not in a dire situation. This precautionary effect is reinforced when the regime shift is irreversible and in particular when it entails a catastrophic consequence. We, on the other hand, showed that even if the risk of regime shifts is endogenous, the precautionary policy is not always guaranteed and the potential regime shifts can even encourage more aggressive resource use when the resource stock is already dwindling. Also, unlike the system in the absence of regime shifts, there can exist a resource-depletion trap in which a regime shift, once it happens, triggers an irreversible process of resource deterioration.

This result has a profound implication especially when the resource management entails strategic interaction among players. For the players' strategic extractions to be precautionary, the remaining stock needs to be relatively large in the first place. If a common-property resource is already mismanaged due to the tragedy of the commons with only a small fraction of resource left, the aggressive resource exploitation under potential regime shifts causes the collapse of the otherwise sustainable resource base. This finding contrasts with those obtained in the existing studies, which focus on the optimal management and the steady-state analysis. Our analysis therefore suggests the importance of the off-steady-state analysis as well as the consideration of strategic interaction.

The model we analyzed could be used for a wide range of problems. As was mentioned in the applied section, the model of a common-property resource problem with a linear production technology allows for several different interpretations, possibly capturing the interaction between endogenous economic growth and natural resource management. If interpreted this way, the findings of this paper could indicate that a sustainable growth and a gradual collapse of economy are both possible under the risk of regime shifts, depending on the current stock of resources. Apart from this particular applied model, the general framework presented here is applicable to other economic issues as well. For instance, one could interpret a regime shift as an innovation that changes the structure of game in a discontinuous manner. The probability of innovation might depend on the stock of public knowledge, which is accumulated through external R&D investment by each player. As shown in this paper, the equilibrium of such a game might be quite different from the one usually expected in deterministic models.

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A Proofs

A.1 Proof of Proposition 1

Fix $\theta \in \Theta$ and let $J(s, z, \{x(t)\}; \theta)$ be the value of the objective function along a feasible path $\{x(t)\}$ from $z(s) = z$. Then for any $\Delta s > 0$,

$$J(s,z,\{x(t)\};\theta) = \int_s^{s+\Delta s} e^{-\rho(t-s)} \Big\{ U(x(t), z(t);\theta) + \lambda(z(t);\theta)\mathbb{E}[V(z(t);\theta')|\theta] \Big\} e^{-\int_s^t \lambda(z(\tau);\theta)d\tau} dt + e^{-\rho\Delta s} e^{-\int_s^{s+\Delta s} \lambda(z_\tau;\theta)d\tau} J(s+\Delta s, z+\Delta z, \{x(t)\};\theta), \quad \text{(A.1)}$$

where we define Δz by

$$\Delta z := \int_s^{s+\Delta s} \dot{z}(t)dt = \int_s^{s+\Delta s} G(z(t), (N-1)\phi(z(t);\theta) + x(t);\theta)dt. \quad \text{(A.2)}$$

By definition, $V(z;\theta) = \max_{\{x(t)\}_{t\geq s}} J(s,z,\{x(t)\};\theta)$ and $V(z+\Delta z;\theta) = \max_{\{x(t)\}_{t\geq s+\Delta s}} J(s+\Delta s, z+\Delta z, \{x(t)\};\theta)$. So we have

$$V(z;\theta) = \max_{\{x(t)\}_{s+\Delta s \geq t \geq s}} \left\{ \int_s^{s+\Delta s} e^{-\rho(t-s)} \{ U(x(t), z(t);\theta) + \lambda(z(t);\theta)\mathbb{E}\left[V(z(t);\theta')|\theta\right] \} e^{-\int_s^t \lambda(z(\tau);\theta)d\tau} dt + e^{-\rho\Delta s} e^{-\int_s^{s+\Delta s} \lambda(z_\tau;\theta)d\tau} V(z+\Delta z;\theta) \right\}, \quad \text{(A.3)}$$

or equivalently

$$0 = \max_{\{x(t)\}_{s+\Delta s \geq t \geq s}} \left\{ \frac{1}{\Delta s} \int_s^{s+\Delta s} e^{-\rho(t-s)} \Big\{ U(x(t), z(t);\theta) + \lambda(z(t);\theta)\mathbb{E}[V(z(t);\theta')|\theta] \Big\} e^{-\int_s^t \lambda(z(\tau);\theta)d\tau} dt + \frac{e^{-\rho\Delta s} e^{-\int_s^{s+\Delta s} \lambda(z_\tau;\theta)d\tau} V(z+\Delta z;\theta) - V(z;\theta)}{\Delta s} \right\}. \quad \text{(A.4)}$$

Since $V(z;\theta)$ is differentiable by assumption, by passing to the limit for $\Delta s \to 0$, we obtain the Hamilton-Jacobi-Bellman equation

$$\rho V(z;\theta) = \max_{x \in X(z)} \Big\{ U(x,z;\theta) - \lambda(z;\theta)V(z;\theta) + \lambda(z;\theta)\mathbb{E}[V(z;\theta')|\theta] + V'(z;\theta)G(z, (N-1)\phi(z;\theta) + x;\theta) \Big\}. \quad \text{(A.5)}$$

Since ϕ is the optimal policy, (2.9) follows.

A.2 Proof of Proposition 2

Fix $\theta \in \Theta_p$. Since V and ϕ satisfy the HJB equation,

$$\begin{aligned}\rho V(z;\theta) &= U(\phi(z;\theta), z;\theta) + V'(z;\theta)G(z, N\phi(z;\theta);\theta) \\ &\quad + \lambda(z;\theta)\left\{\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta)\right\} && \text{(A.6)} \\ &\geq U(x, z;\theta) + V'(z;\theta)G(z,(N-1)\phi(z;\theta)+x;\theta) \\ &\quad + \lambda(z;\theta)\left\{\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta)\right\} && \text{(A.7)}\end{aligned}$$

for any $x \in X(z)$. Let $\{\hat{x}(t)\}$ be an arbitrary feasible path from z and $\{\hat{z}(t)\}$ be the associated path of state variable so that $\hat{x}(t) \in X(\hat{z}(t))$ for all $t \in [s,\infty)$ and $\dot{\hat{z}}(t) = G(\hat{z}(t),(N-1)\phi(\hat{z}(t);\theta) + \hat{x}(t);\theta)$ with $\hat{z}(s) = z$. Then for all $t \in [s,\infty)$

$$\begin{aligned}\rho V(\hat{z}(t);\theta) \geq\ & U(\hat{x}(t),\hat{z}(t);\theta) + V'(\hat{z}(t);\theta)G(\hat{z}(t),(N-1)\phi(\hat{z}(t);\theta)+\hat{x}(t);\theta) \\ & + \lambda(\hat{z}(t);\theta)\left\{\mathbb{E}[V(\hat{z}(t);\theta')|\theta] - V(\hat{z}(t);\theta)\right\} \qquad \text{(A.8)}\end{aligned}$$

while

$$\begin{aligned}\rho V(z(t);\theta) =\ & U(\phi(z(t)), z(t);\theta) + V'(z(t);\theta)G(z(t), N\phi(z(t);\theta);\theta) \\ & + \lambda(z(t);\theta)\left\{\mathbb{E}[V(z(t);\theta')|\theta] - V(z(t);\theta)\right\} \qquad \text{(A.9)}\end{aligned}$$

where $\{z(t)\}$ satisfies $\dot{z}(t) = G(z(t), N\phi(z(t);\theta);\theta)$ with $z(s) = z$.

Let $J_T(s,z,\{\hat{x}(t)\};\theta)$ and $J_T(s,z,\phi_\theta;\theta)$ be the values of objective function along $\{\hat{x}(t)\}$ and $\{\phi(z(t);\theta)\}$, respectively, truncated up until $T > s$. From (A.8)

$$\begin{aligned}J_T(s,z,\{\hat{x}(t)\};\theta) &\leq \int_s^T e^{-\rho(t-s)}\Big\{\rho V(\hat{z}(t);\theta) - V'(\hat{z}(t);\theta)\dot{\hat{z}}(t) \\ &\qquad + \lambda(\hat{z}(t);\theta)V(\hat{z}(t);\theta)\Big\}e^{-\int_s^t \lambda(\hat{z}(\tau);\theta)d\tau}dt && \text{(A.10)} \\ &= -\int_s^T \frac{d}{dt}\left\{e^{-\rho(t-s)}V(\hat{z}(t);\theta)e^{-\int_s^t \lambda(\hat{z}(\tau);\theta)d\tau}\right\}dt && \text{(A.11)} \\ &= V(z;\theta) - e^{-\rho(T-s)}V(\hat{z}(T);\theta)e^{-\int_s^T \lambda(\hat{z}(\tau);\theta)d\tau} && \text{(A.12)}\end{aligned}$$

and from (A.9)

$$\begin{aligned}J_T(s,z,\phi_\theta;\theta) &= \int_s^T e^{-\rho(t-s)}\Big\{\rho V(z(t);\theta) - V'(z(t);\theta)\dot{z}(t) \\ &\qquad + \lambda(z(t);\theta)V(z(t);\theta)\Big\}e^{-\int_s^t \lambda(z(\tau);\theta)d\tau}dt && \text{(A.13)} \\ &= V(z;\theta) - e^{-\rho(T-s)}V(z(T);\theta)e^{-\int_s^T \lambda(z(\tau);\theta)d\tau}. && \text{(A.14)}\end{aligned}$$

Therefore

$$\begin{aligned}& J_T(s,z,\phi_\theta;\theta) - J_T(s,z,\{\hat{x}(t)\};\theta) \\ &\quad \geq e^{-\rho(T-s)}\left[V(\hat{z}(T);\theta)e^{-\int_s^T \lambda(\hat{z}(\tau);\theta)d\tau} - V(z(T);\theta)e^{-\int_s^T \lambda(z(\tau);\theta)d\tau}\right] \qquad \text{(A.15)}\end{aligned}$$

for all $T > s$. If (2.12) is satisfied, then

$$\lim_{T\to\infty} J_T(s,z,\phi_\theta;\theta) - \lim_{T\to\infty} J_T(s,z,\{\hat{x}(t)\};\theta) \geq 0 \tag{A.16}$$

for any feasible path $\{\hat{x}(t)\}$, which implies that ϕ solves the problem (2.4). Finally, (A.14) with (2.12) shows $\lim_{T\to\infty} J_T(s,z,\phi_\theta;\theta) = V(z;\theta)$, which completes the proof.

A.3 Proof of Proposition 3

Suppose $V(z;\theta) = \gamma(\theta)z$ for some constant $\gamma(\theta)$. Then the HJB equation is given by

$$\rho\gamma(\theta) = u(\beta(\theta);\theta) + \gamma(\theta)[R - N\beta(\theta)], \tag{A.17}$$

where $\beta(\theta) := v(\gamma(\theta);\theta)$. Rearranging the terms yields $H(\gamma(\theta);\theta) = 0$. Then by definition of $\gamma^*(\theta)$ and $\beta^*(\theta)$, $V(z;\theta) := \gamma^*(\theta)z$ and $\phi(z;\theta) := \beta^*(\theta)z$ satisfy the HJB equation. Comparative results in terms of parameters R, ρ, ν, and N are immediate from (3.4) given the definition of u and v. Moreover, since $\dot{z}(t)/z(t) \leq R$ for any feasible path, we have

$$0 \leq e^{-\rho(T-s)}V(z(T);\theta) = e^{-\rho(T-s)}\gamma^*(\theta)z(T) \leq \gamma^*(\theta)z(s)e^{-(\rho-R)(T-s)} \tag{A.18}$$

for any $T \geq s$. Notice that $\rho > R$ by assumption. Therefore, the right-hand side of the second inequality converges to 0 as $T \to \infty$. Since $e^{-\int_s^T \lambda(z(\tau);\theta)d\tau}$ is bounded, the transversality condition is also satisfied.

A.4 Proof of Proposition 4

Let $\lambda(\theta) := \lambda(z;\theta)$. Suppose for each $\theta \in \Theta_p$ there exists some constant $\gamma(\theta)$ such that $V(z;\theta) = \gamma(\theta)z$ for every $z \in \mathbb{R}_+$. Then the HJB equation is given by

$$\rho\gamma(\theta) + \left(\gamma(\theta) - \mathbb{E}[\gamma(\theta')|\theta]\right)\lambda(\theta) = u(\beta(\theta);\theta) - \gamma(\theta)[R - N\beta(\theta)], \tag{A.19}$$

where $\beta(\theta) := v(\gamma(\theta);\theta)$. Rearranging the terms yields

$$\gamma(\theta) - \mathbb{E}[\gamma(\theta')|\theta] = \frac{H(\gamma(\theta);\theta)}{\lambda(\theta)}. \tag{A.20}$$

Note that $\dot{z}/z \leq R$ for any feasible path, $\lambda(\theta) > 0$, and $\rho > R$. So the transversality condition is satisfied for any value function of the form $V(z;\theta) = \gamma(\theta)z$. Hence, if there exists a solution $\{\gamma^x(\theta)\}_{\theta\in\Theta_p}$ to (A.19), then $V(z;\theta) = \gamma^x(\theta)z$ and $\phi(z;\theta) = \beta^x(\theta)z$ where $\beta^x(\theta) := v(\gamma^x(\theta);\theta)$ constitute an MPNE.

We shall show that there exists a solution to (A.19). By assumption, support Θ_p of p is finite. So we may write $\Theta_p = \{\theta_1, \theta_2, \ldots, \theta_M\}$ for some $M \in \mathbb{N}$. Let

$\gamma_m := \gamma(\theta_m)$ for each $m \in \{1, 2, \ldots, M\}$ and $p_{m'|m} := p(\theta_{m'}|\theta_m)$. Then (A.20) may be written as a system of M equations

$$\gamma_m = \sum_{m' \neq m} \frac{p_{m'|m}}{1 - p_{m|m}} \gamma_{m'} + \frac{[\lambda(\theta_m)]^{-1}}{1 - p_{m|m}} H(\gamma_m; \theta_m), \quad m = 1, 2, \ldots, M. \tag{A.21}$$

Observe that $\lim_{\gamma \to 0} H(\gamma; \theta) = u(v(0; \theta); \theta) > 0$ and $\lim_{\gamma \to \infty} H(\gamma; \theta) = -\infty$, which means that for each $\gamma_{-m} := (\gamma_1, \ldots, \gamma_{m-1}, \gamma_{m+1}, \ldots, \gamma_M) \in \mathbb{R}_+^{M-1}$, there exists $\gamma_m > 0$ that satisfies (A.21). Moreover, Appendix B.3 shows that $\partial H(\gamma; \theta)/\partial \gamma < 0$ for all $\gamma > 0$, implying that such a γ_m must be unique. Since H is continuously differentiable with respect to γ, the implicit function theorem then suggests that there exist a continuously differentiable function $\Gamma_m : \mathbb{R}_+^{M-1} \to \mathbb{R}_{++}$ such that

$$\Gamma_m(\gamma_{-m}) = \sum_{m' \neq m} \frac{p_{m'|m}}{1 - p_{m|m}} \gamma_{m'} + \frac{[\lambda(\theta_m)]^{-1}}{1 - p_{m|m}} H(\Gamma_m(\gamma_{-m}); \theta_m) \tag{A.22}$$

for any $\gamma_{-m} \in \mathbb{R}_+^{M-1}$. Define $\mathbf{\Gamma} : \mathbb{R}_{++}^M \to \mathbb{R}_{++}^M$ by

$$\mathbf{\Gamma}(\gamma_1, \ldots, \gamma_M) := (\Gamma_1(\gamma_{-1}), \ldots, \Gamma_M(\gamma_{-M})), \tag{A.23}$$

of which we shall find a fixed point.

Let $\overline{\gamma}^* := \max_m\{\gamma^*(\theta_m)\}$ and $\underline{\gamma}^* := \min_m\{\gamma^*(\theta_m)\}$. Since H is strictly decreasing and since γ_m^* is the unique solution to $H(\gamma; \theta_m) = 0$,

$$H(\overline{\gamma}^*; \theta_m) \leq 0 \leq H(\underline{\gamma}^*; \theta_m) \tag{A.24}$$

for all m. By putting $\mathbf{1} := (1, \ldots, 1) \in \mathbb{R}_{++}^{M-1}$, we have

$$\Gamma_m(\mathbf{1} \cdot \underline{\gamma}^*) = \underline{\gamma}^* + \frac{[\lambda(\theta_m)]^{-1}}{1 - p_{m|m}} H(\Gamma_m(\mathbf{1} \cdot \underline{\gamma}^*); \theta_m) \geq \underline{\gamma}^* \tag{A.25}$$

and

$$\Gamma_m(\mathbf{1} \cdot \overline{\gamma}^*) = \overline{\gamma}^* + \frac{[\lambda(\theta_m)]^{-1}}{1 - p_{m|m}} H(\Gamma_m(\mathbf{1} \cdot \overline{\gamma}^*); \theta_m) \leq \overline{\gamma}^* \tag{A.26}$$

for all m. Since $\partial \Gamma_m(\gamma_{-m}; \theta)/\partial \gamma_{m'} > 0$ for every $m' \neq m$, this implies

$$\mathbf{\Gamma}(\gamma_1, \ldots, \gamma_M) \in \bigtimes_{m=1}^{M} [\underline{\gamma}^*, \overline{\gamma}^*] \quad \forall (\gamma_1, \ldots, \gamma_M) \in \bigtimes_{m=1}^{M} [\underline{\gamma}^*, \overline{\gamma}^*]. \tag{A.27}$$

By Browder's fixed point theorem, there exists $(\gamma_1^x, \ldots, \gamma_M^x)$ such that $\gamma_m^x \in [\underline{\gamma}^*, \overline{\gamma}^*]$ for all m and $\mathbf{\Gamma}(\gamma_1^x, \ldots, \gamma_M^x) = (\gamma_1^x, \ldots, \gamma_M^x)$. This fixed point constitutes a solution to (A.21) and thus to (A.19).

To see the latter part of the proposition, let $\bar{m} \in \operatorname{argmax}\{\gamma_m^*\}$ so that $\gamma_{\bar{m}}^* = \overline{\gamma}^*$. Suppose by way of contradiction $\gamma_{\bar{m}}^x \geq \gamma_{\bar{m}}^*$. Then $H(\gamma_{\bar{m}}^x;\theta_{\bar{m}}) \leq H(\gamma_{\bar{m}}^*;\theta_{\bar{m}}) = 0$ and thus

$$\gamma_{\bar{m}}^* \leq \gamma_{\bar{m}}^x = \sum_{m' \neq \bar{m}} \frac{p_{m'|\bar{m}}}{1 - p_{\bar{m}|\bar{m}}} \gamma_{m'}^x + \frac{[\lambda(\theta_{\bar{m}})]^{-1}}{1 - p_{\bar{m}|\bar{m}}} H(\gamma_{\bar{m}}^x;\theta_{\bar{m}}) \leq \sum_{m' \neq \bar{m}} \frac{p_{m'|\bar{m}}}{1 - p_{\bar{m}|\bar{m}}} \gamma_{m'}^x. \tag{A.28}$$

Now that Θ_p consists of strictly distinct regimes, there exists m such that $\gamma_m^* < \overline{\gamma}^*$. For such m, it must be the case that $H(\overline{\gamma}^*;\theta_m) < H(\gamma_m^*;\theta_m) = 0$ because H is strictly decreasing. Since $\gamma_{m'}^x \leq \overline{\gamma}^*$ for all m', it then follows that

$$\gamma_m^x = \sum_{m' \neq m} \frac{p_{m'|m}}{1 - p_{m|m}} \gamma_{m'}^x + \frac{[\lambda(\theta_m)]^{-1}}{1 - p_{m|m}} H(\gamma_m^x;\theta_m) < \overline{\gamma}^*. \tag{A.29}$$

(A.28) and (A.29) then imply $\gamma_{\bar{m}}^* < \overline{\gamma}^*$, a contradiction. Therefore $\gamma_{\bar{m}}^x < \gamma_{\bar{m}}^*$ and

$$\beta^x(\theta_{\bar{m}}) = v(\gamma^x(\theta_{\bar{m}});\theta_{\bar{m}}) > v(\gamma^*(\theta_{\bar{m}});\theta_{\bar{m}}) = \beta^*(\theta_{\bar{m}}). \tag{A.30}$$

A.5 Proof of Corollary 1

Let $\gamma_1^* := \gamma^*(\theta_1)$ and $\gamma_2^* := \gamma^*(\theta_2)$. By Proposition 3 we know $\gamma_1^* > \gamma_2^*$ because $\gamma(\theta)$ is increasing in R and ν, and decreasing in ρ. Let $\gamma_1^x := \gamma^x(\theta_1)$, $\gamma_2^x := \gamma^x(\theta_2)$. Then combining the HJB equations yields

$$\frac{1}{p_{2|1}\lambda(\theta_1)} H(\gamma_1^x;\theta_1) = \gamma_1^x - \gamma_2^x = -\frac{1}{p_{1|2}\lambda(\theta_2)} H(\gamma_2^x;\theta_2). \tag{A.31}$$

Since $H(\gamma_1^*;\theta_1) = H(\gamma_2^*;\theta_2) = 0$ and H is strictly decreasing,

$$\gamma_1^x \geq \gamma_1^* \iff \gamma_1^x \leq \gamma_2^x \iff \gamma_2^* \geq \gamma_2^x. \tag{A.32}$$

Therefore $\gamma_2^* < \gamma_2^x < \gamma_1^x < \gamma_1^*$, and thus $\beta^x(\theta_1) > \beta^*(\theta_1)$ and $\beta^x(\theta_2) < \beta^*(\theta_2)$.

A.6 Proof of Proposition 5

For $m, m' \in \{1,2\}$ with $m' \neq m$, the HJB equation is given by

$$\begin{aligned}(\rho_m + p_{m'|m}\lambda(z;\theta_m))V(z;\theta_m) - p_{m'|m}\lambda(z;\theta_m)V(z;\theta_{m'}) \\ = \left\{u(\beta(z;\theta_m);\theta_m) + u'(\beta(z;\theta_m);\theta_m)(R_m - N\beta(z;\theta_m))\right\} z,\end{aligned} \tag{A.33}$$

where $\beta(z;\theta) := v(V'(z;\theta);\theta)$. Notice that $V'(z;\theta_m)$ is already eliminated by using the first-order condition $u'(\beta(z;\theta_m);\theta_m) = V'(z;\theta_m)$. Also note that $\lambda(z;\theta_1) = \lambda(z;\theta_2) = 0$ for $z \geq \bar{z}$, meaning that the HJB equations coincide with those in the absence of regime shift. Hence, by Proposition 3, we know that $V(z;\theta) = \gamma^*(\theta)z$ and $\phi(z;\theta) = \beta^*(\theta)z$ with $\beta^*(\theta) = v(\gamma^*(\theta);\theta)$ solve the HJB equations for all $z \geq \bar{z}$.

To find a solution for $z < \bar{z}$, differentiate (A.33) with respect to z, eliminate V', and then rearrange the terms to obtain the system of differential equations

$$[\beta'(z;\theta_1), \beta'(z;\theta_2)] = [B_1(z, \beta(z;\theta_1), \beta(z;\theta_2)), B_2(z, \beta(z;\theta_1), \beta(z;\theta_2))], \quad \text{(A.34)}$$

where $\beta' := \partial\beta/\partial z$ and

$$B_m(z, \beta_m, \beta_{m'}) := \frac{1}{F_m(\beta_m)} \left\{ \Lambda_m^1(z)\hat{H}_m(\beta_m) - \Lambda_m^2(z)\hat{H}_{m'}(\beta_{m'}) + \Lambda_m^3(z)\Delta_m(\beta_m) \right\}, \quad \text{(A.35)}$$

$$F_m(\beta) := -\left[H'(u'(\beta;\theta_m);\theta_m) + \rho_m\right] u''(\beta;\theta_m) \quad \text{(A.36)}$$

$$\Lambda_m^1(z) := \frac{1}{z} - \frac{p_{m'|m}\lambda'(z;\theta_m)\rho_{m'}}{\varphi(z)}, \quad \Lambda_m^2(z) := -\frac{p_{m'|m}\lambda'(z;\theta_m)\rho_m}{\varphi(z)}, \quad \text{(A.37)}$$

$$\Lambda_m^3(z) := \frac{p_{m'|m}\lambda'(z;\theta_m)\rho_m\rho_{m'}}{\varphi(z)} + \frac{p_{m'|m}\lambda(z;\theta_m)}{z}, \quad \text{(A.38)}$$

$$\hat{H}_m(\beta) := H(u'(\beta;\theta_m);\theta_m), \quad \text{(A.39)}$$

$$\Delta_m(\beta) := u'(\beta;\theta_{m'}) - u'(\beta;\theta_m), \quad \text{(A.40)}$$

and $\varphi(z) := \rho_1\rho_2 + \rho_2 p_{2|1}\lambda(z;\theta_1) + \rho_1 p_{1|2}\lambda(z;\theta_2)$. Appendix B.4 shows that $F_m(\beta) > 0$ so the right-hand side of (A.34) is well defined and continuously differentiable on $D := (0, \bar{z}] \times \mathbb{R}^2_{++}$. Then, by Peano's existence theorem, for each $(z, \beta) \in D$ there exists the unique local solution. Let $(\beta^l(z;\theta_1), \beta^l(z;\theta_2))$ be a solution of (A.34) such that $\beta^l(\bar{z};\theta) = \beta^*(\theta)$ for each $\theta \in \{\theta_1, \theta_2\}$. Going backward from $\bar{z}$, we obtain the trajectory of this solution satisfying the HJB equations for $z \leq \bar{z}$ as well.

To characterize the equilibrium, notice that since $u'(\beta^*(\theta);\theta) = \gamma^*(\theta)$,

$$\hat{H}_m(\beta^l(\bar{z};\theta_m)) = H(u'(\beta^*(\theta_m);\theta_m);\theta_m) = H(\gamma^*(\theta_m);\theta_m) = 0, \quad \text{(A.41)}$$

and

$$\Delta_m(\beta^l(\bar{z};\theta_m)) = u'(\beta^*(\theta_{m'});\theta_m) - u'(\beta^*(\theta_m);\theta_{m'}) = \gamma^*(\theta_{m'}) - \gamma^*(\theta_m). \quad \text{(A.42)}$$

Combining these equations, we obtain

$$\lim_{z \nearrow \bar{z}} \frac{\partial \beta^l(z;\theta_m)}{\partial z} = \frac{p_{m'|m} \lim_{z \nearrow \bar{z}} \lambda'(z;\theta_m)}{F_m(\beta^*(\theta_m))} [\gamma^*(\theta_{m'}) - \gamma^*(\theta_m)]. \quad \text{(A.43)}$$

Since $\gamma^*(\theta_2) - \gamma^*(\theta_1) < 0$ by assumption, $\lim_{z \nearrow \bar{z}} \lambda'(z;\theta_1) < 0$ implies that $\beta^l(z;\theta_1)$ always converges to $\beta^*(\theta_1)$ from below as $z \to \bar{z}$ from left. On the other hand, $\lim_{z \nearrow \bar{z}} \lambda'(z;\theta_2) < 0$ implies that $\beta^l(z;\theta_2)$ always converges to $\beta^*(\theta_2)$ from above. The assertions of the proposition then follow from the continuity of $\beta^l(z;\theta)$ at $\bar{z}$.

A.7 Proof of Proposition 6

First consider the equilibrium extraction rate in regime θ_2 for each game. For game $\langle N, U, G, \lambda, \Theta, p\rangle$, Proposition 5 suggests that there exists $z_2^\star < \bar{z}$ such that $\beta_p^l(z;\theta) > \beta^*(\theta_2)$ and thus $\hat{H}_2(\beta_p^l(z;\theta_2)) > 0$ for all $z \in (z_2^\star, \bar{z})$. For game $\langle N, U, G, \lambda, \Theta, \hat{p}\rangle$, on the other hand, there is no risk of regime shift in θ_2 because $\hat{p}(\theta_1|\theta_2) = 0$. The HJB equation in regime θ_2 thus coincides with that in the absence of regime shift for all z. This means $\beta_{\hat{p}}^l(z;\theta_2) = \beta^*(\theta_2)$ and thus $\hat{H}_2(\beta_{\hat{p}}^l(z;\theta_2)) = 0$ for all z. Also notice that since $\lambda(z;\theta)$ is continuously differentiable and since $\lim_{z\nearrow\bar{z}} \lambda'(z;\theta) < 0$, there exists $\underline{z} < \bar{z}$ such that $\lambda'(z;\theta) < 0$ for all $z \in [\underline{z}, \bar{z}]$. Therefore,

$$\Lambda_2^2(z)\hat{H}_2(\beta_{\hat{p}}^l(z;\theta_2)) = 0 < \Lambda_2^2(z)\hat{H}_2(\beta_p^l(z;\theta_2)) \tag{A.44}$$

for all $z \in (\tilde{z}, \bar{z})$ where $\tilde{z} := \max\{z_2^\star, \underline{z}\} < \bar{z}$.

Let us now turn to the equilibrium extraction rate in regime θ_1 for each game. By the same argument as in Proposition 5, we can see that the trajectories of $\beta_p^l(z;\theta_1)$ and $\beta_{\hat{p}}^l(z;\theta_1)$ are characterized by (A.34) and (A.35) and

$$\lim_{z\nearrow\bar{z}} \frac{\partial\beta_{\hat{p}}^l(z;\theta_1)}{\partial z} = \lim_{z\nearrow\bar{z}} \frac{\partial\beta_p^l(z;\theta_1)}{\partial z} > 0. \tag{A.45}$$

Since $\beta_p^l(\bar{z};\theta_1) = \beta_{\hat{p}}^l(\bar{z};\theta_1) = \beta^*(\theta_1)$, it follows from (A.34), (A.35), (A.44), and (A.45) that there exists $\hat{z} < \bar{z}$ such that for all $z \in (\hat{z}, \bar{z})$

$$\frac{\partial\beta_{\hat{p}}^l(z;\theta_1)}{\partial z} > \frac{\partial\beta_p^l(z;\theta_1)}{\partial z} > 0, \tag{A.46}$$

meaning that the slope of $\beta_{\hat{p}}^l(z;\theta_1)$ is steeper than that of $\beta_p^l(z;\theta_1)$ in a neighborhood of $\bar{z}$ excluding $\bar{z}$. Since both trajectories converge to the same value from below as z approaches to $\bar{z}$, this implies $\beta_{\hat{p}}^l(z;\theta_1) < \beta_p^l(z;\theta_1)$ at least in that interval.

A.8 Proof of Proposition 7

By the same argument as in Proposition 6, we obtain

$$\lim_{z\nearrow\bar{z}} \frac{\partial\beta_{\hat{p}}^l(z;\theta_1)}{\partial z} = \frac{\hat{p}_{2|1} \lim_{z\nearrow\bar{z}} \lambda'(z;\theta_1)}{F_1(\beta^*(\theta_1))}[\gamma^*(\theta_2) - \gamma^*(\theta_1)], \tag{A.47}$$

$$\lim_{z\nearrow\bar{z}} \frac{\partial\beta_{\hat{p}^c}^l(z;\theta_1)}{\partial z} = \frac{\hat{p}_{2|1}^c \lim_{z\nearrow\bar{z}} \lambda'(z;\theta_1)}{F_1(\beta^*(\theta_1))}[\gamma^*(\theta_2^c) - \gamma^*(\theta_1)]. \tag{A.48}$$

Since $\hat{p}_{2|1} = \hat{p}_{2|1}^c > 0$, $\gamma^*(\theta_1) > \gamma^*(\theta_2) > \gamma^*(\theta_2^c) = 0$, and $\lim_{z\nearrow\bar{z}} \lambda'(z;\theta_1) < 0$, this implies $\lim_{z\nearrow\bar{z}} \partial\beta_{\hat{p}^c}^l(z;\theta_1)/\partial z > \lim_{z\nearrow\bar{z}} \partial\beta_{\hat{p}}^l(z;\theta_1)/\partial z > 0$, meaning that the slope of $\beta_{\hat{p}^c}^l(z;\theta_1)$ is steeper than that of $\beta_{\hat{p}}^l(z;\theta_1)$ in a neighborhood of $\bar{z}$. Therefore, the assertion of the proposition follows from the continuity of β^l.

B Computations

B.1 Uniqueness of the MPNE

Note that $\lim_{\gamma\to 0} H(\gamma;\theta) = \nu(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\kappa^{-\frac{\alpha}{1-\alpha}} > 0$ and $\lim_{\gamma\to\infty} H(\gamma;\theta) = -\infty$. Since $H(\cdot;\theta)$ is continuous on $[0,\infty)$, this means that $H(\gamma;\theta) = 0$ has at least one solution and the solutions must be strictly positive. Let $\gamma^*(\theta)$ be a solution. Denote $H' := \partial H/\partial\gamma$ and observe

$$H'(\gamma;\theta) = \left[\frac{\kappa}{\kappa+\gamma} - \alpha\right]\frac{H(\gamma;\theta)}{(1-\alpha)\gamma} - \frac{(\rho-R)\gamma}{(1-\alpha)(\kappa+\gamma)} - \frac{(1-\alpha)\kappa}{\alpha\gamma}v(\gamma;\theta), \quad \text{(B.1)}$$

Since $H(\gamma^*(\theta);\theta) = 0$, $\gamma^*(\theta) > 0$, and $v(\gamma;\theta) > 0$, we have $H'(\gamma^*(\theta);\theta) < 0$. This means that the function H always crosses zero from above. The uniqueness of the solution then follows from the continuity of H.

B.2 Inefficiency of the MPNE

Since $\gamma^*(\theta)$ and $\beta^*(\theta)$ satisfy the HJB equation (A.17),

$$u(\beta^*(\theta);\theta) = \gamma^*(\theta)(\rho - R + N\beta^*(\theta)) = u'(\beta^*(\theta);\theta)(\rho - R + N\beta^*(\theta)), \quad \text{(B.2)}$$

where the last equality follows from the first-order condition $u'(\beta^*(\theta);\theta) = \gamma^*(\theta)$. Thus

$$\begin{aligned}\left.\frac{\partial\tilde{V}(\beta;\theta)}{\partial\beta}\right|_{\beta=\beta^*(\theta)} &= \frac{\tilde{V}(\beta^*(\theta);\theta)}{\rho - R + N\beta^*(\theta)}\left\{\frac{(\rho - R + N\beta^*(\theta))u'(\beta^*(\theta);\theta)}{u(\beta^*(\theta);\theta)} - N\right\}\\ &= \frac{\gamma^*(\theta)z}{\rho - R + N\beta^*(\theta)}(1-N), \end{aligned}\quad \text{(B.3)}$$

which is negative if and only if $N > 1$.

B.3 Monotonicity of H function

Note that

$$H'(\gamma;\theta) = -(\rho - R) - \frac{v(\gamma;\theta)}{1-\alpha}\left[1 - \alpha N + (N-1)\frac{\kappa}{\kappa+\gamma}\right] \quad \text{(B.4)}$$

therefore

$$\lim_{\gamma\to 0} H'(\gamma;\theta) = -(\rho - R) - Nv(0;\theta) < -(\rho - R) = \lim_{\gamma\to\infty} H'(\gamma;\theta). \quad \text{(B.5)}$$

Letting $H'' := \partial^2 H/\partial\gamma^2$, we also note

$$H''(\gamma;\theta) = \frac{v(\gamma;\theta)}{(1-\alpha)(\kappa+\gamma)^2}\left[(2N-1)\kappa + \frac{1-\alpha N}{1-\alpha}\gamma\right]. \quad \text{(B.6)}$$

Hence

$$\sup H'(\gamma;\theta) = \begin{cases} -\rho + R < 0 & \text{if } \alpha N \leq 1, \\ H'(\tilde{\gamma}(\theta);\theta) & \text{otherwise,} \end{cases} \tag{B.7}$$

where

$$\tilde{\gamma}(\theta) := \frac{(2N-1)(1-\alpha)\kappa}{\alpha N - 1}. \tag{B.8}$$

Suppose $\alpha N > 1$. A tedious computation then yields

$$H'(\tilde{\gamma}(\theta);\theta) = (\rho - R) \left\{ \left[\kappa^{-1} \frac{\alpha}{N-1} \left(\frac{\nu}{\rho - R} \right)^{1-\alpha} \left(\frac{\alpha N - 1}{2 - \alpha} \right)^{2-\alpha} \right]^{\frac{1}{1-\alpha}} - 1 \right\}, \tag{B.9}$$

which is strictly negative since $\kappa > \underline{\kappa}(\theta)$. Therefore, H is monotonically decreasing.

B.4 Positivity of F function

The discussion in Appendix B.3 suggests

$$\min_{\beta} \left[H'(u'(\beta;\theta);\theta) - \rho \right] = \min_{\gamma} \left[H'(\gamma;\theta) - \rho \right] = R - N\nu(\alpha/\kappa)^{\frac{1}{1-\alpha}}, \tag{B.10}$$

which is strictly positive because $\kappa > \underline{\kappa}(\theta)$. Since $u''(\beta;\theta) < 0$ for all β, this then implies

$$F(\beta;\theta) := -\left[H'(u'(\beta;\theta);\theta) - \rho \right] u''(\beta;\theta) > 0. \tag{B.11}$$

C More on the condition for precautionary policy (not for publication)

In the main text, we identified that the sign of

$$\frac{d}{dz} \left\{ \lambda(z;\theta) \left(\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta) \right) \right\} \Big|_{z=z_{ss}} \tag{C.1}$$

determines whether precautionary policy emerges at some stationary point $z = z_{ss}$. In this appendix, we show that this condition is consistent with the existing results by demonstrating that it is in fact applicable to other models of regime shifts.

C.1 Application to Polasky et al. (2011)

Consider first a game $\langle N, U, G, \lambda, \Theta, p \rangle$ where $N = 1$,

$$U(x, z; \theta) := qx, \tag{C.2}$$

and

$$G(z, x; \theta) := g(z; \theta) - x, \tag{C.3}$$

with a constraint $0 \leq x \leq x_{max}$ for an arbitrarily large upper bound $x_{max} > 0$. Here $q > 0$ is a parameter that represents the unit price of harvested resource x. This is a slightly generalized version of the model considered by Polasky et al. (2011) in the sense that endless switching among multiple regimes is allowed. We assume that $g(\cdot; \theta)$ satisfies the properties stated in Polasky et al. (2011) for all $\theta \in \Theta_p$. For this model, the function $\Xi(z; \theta)$ boils down to

$$\Xi(z; \theta) = (\rho - g'(z; \theta))q, \tag{C.4}$$

which is increasing in z at least in a neighborhood of the steady state.

To see that the sign of (C.1) in fact provides the condition for a precautionary policy, let us consider a bang-bang control policy

$$\phi(z; \theta) := \begin{cases} 0 & \text{for } z < z_{ss}(\theta) \\ g(z; \theta) & \text{for } z = z_{ss}(\theta) \\ x_{max} & \text{for } z > z_{ss}(\theta) \end{cases} \tag{C.5}$$

where $z_{ss}(\theta)$ is the steady-state level of resource stock under regime θ. If this constitutes an equilibrium, then the value function $V(\cdot; \theta)$ must satisfy the HJB equation, which may be written as

$$\begin{aligned} &\rho V(z; \theta) - \lambda(z; \theta) \left(\mathbb{E}[V(z; \theta') | \theta] - V(z; \theta) \right) \\ &= \begin{cases} V'(z; \theta) g(z; \theta) & \text{for } z < z_{ss}(\theta) \\ q g(z; \theta) & \text{for } z = z_{ss}(\theta) \\ q x_{max} + V'(z; \theta)[g(z; \theta) - x_{max}] & \text{for } z > z_{ss}(\theta). \end{cases} \end{aligned} \tag{C.6}$$

Suppose $V(\cdot; \theta)$ is continuously differentiable for all $\theta \in \Theta_p$ and differentiate the HJB equation to obtain

$$\begin{aligned} &(\rho - g'(z; \theta)) V'(z; \theta) - \frac{d}{dz} \left\{ \lambda(z; \theta) \left(\mathbb{E}[V(z; \theta') | \theta] - V(z; \theta) \right) \right\} \\ &= \begin{cases} V''(z; \theta) g(z; \theta) & \text{for } z < z_{ss}(\theta) \\ V''(z; \theta)[g(z; \theta) - x_{max}] & \text{for } z > z_{ss}(\theta) \end{cases} \end{aligned} \tag{C.7}$$

If (C.5) is optimal (at least in a neighborhood of $z = z_{ss}(\theta)$), $\phi(\cdot, \theta)$ must satisfy

$$\phi(z;\theta) \in \underset{x \in [0, x_{max}]}{\operatorname{argmax}} \left\{ qx - V'(z;\theta)x \right\}. \tag{C.8}$$

It follows that $V'(z;\theta)$ crosses q at $z = z_{ss}(\theta)$ from above, which together with (C.7) requires $\lim_{z \nearrow z_{ss}(\theta)} V''(z;\theta) = \lim_{z \searrow z_{ss}(\theta)} V''(z;\theta) = 0$. Therefore (C.7) implies

$$\rho > g'(z_{ss}(\theta);\theta) \iff \frac{d}{dz} \left\{ \lambda(z;\theta) \left(\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta) \right) \right\} \Big|_{z = z_{ss}(\theta)} > 0. \tag{C.9}$$

Since the steady-state level $z^*_{ss}(\theta)$ in the absence of a regime shift is determined by $\rho = g'(z^*_{ss}(\theta);\theta)$, it in turn follows that

$$z_{ss}(\theta) > z^*_{ss}(\theta) \iff \frac{d}{dz} \left\{ \lambda(z;\theta) \left(\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta) \right) \right\} \Big|_{z = z_{ss}(\theta)} > 0. \tag{C.10}$$

Notice that the derivation of (C.1) in the main text assumes the optimal policy is an interior solution, which is not the case here. But even in this case, condition (C.1) applies.

C.2 Application to de Zeeuw and Zemel (2012)

Consider another game $\langle N, U, G, \lambda, \Theta, p \rangle$ where $N = 1$,

$$U(x, z;\theta) := bx - \frac{1}{2}x^2 - \frac{c}{2}z^2, \tag{C.11}$$

and

$$G(z, x;\theta) := x - az. \tag{C.12}$$

Here a, b, and c are parameters included in θ. This is a slightly generalized version of the model considered by de Zeeuw and Zemel (2012) because endless switching among multiple regimes is allowed. We also note that here z is not the stock of resources, but the stock of pollution. For this model, the function $\Xi(z;\theta)$ boils down to

$$\Xi(z;\theta) = (\rho + a)(az - b) + cz, \tag{C.13}$$

which is increasing in z.

If $\phi(\cdot;\theta)$ and $V(\cdot;\theta)$ constitute an equilibrium, they must satisfy the HJB equation

$$\rho V(z;\theta) = U(\phi(z;\theta), z;\theta) + V'(z;\theta) \left\{ \phi(z;\theta) - az \right\} + \lambda(z;\theta) \left(\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta) \right), \tag{C.14}$$

and the first-order condition

$$b - \phi(z;\theta) = -V'(z;\theta). \tag{C.15}$$

Differentiating the HJB equation and then eliminating $V'(\cdot;\theta)$ by (C.15) yields

$$\begin{aligned}(\rho + a)(\phi(z;\theta) - b) + cz = V''(z;\theta)\{\phi(z;\theta) - az\} \\ + \frac{d}{dz}\left\{\lambda(z;\theta)\left(\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta)\right)\right\}.\end{aligned} \tag{C.16}$$

Let $z_{ss}(\theta)$ be the steady-state level of stock which is implicitly defined by

$$G(z_{ss}(\theta), \phi(z_{ss}(\theta);\theta);\theta) = \phi(z_{ss}(\theta);\theta) - az_{ss}(\theta) = 0. \tag{C.17}$$

Then (C.16) and (C.17) imply

$$z_{ss}(\theta) - z^*_{ss}(\theta) = \frac{1}{a(\rho + a) + c}\frac{d}{dz}\left\{\lambda(z;\theta)\left(\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta)\right)\right\}\Big|_{z=z_{ss}(\theta)}, \tag{C.18}$$

where $z^*_{ss}(\theta) := (\rho + a)b/[a(\rho + a) + c]$ is the steady-state level of stock in the absence of a regime shift. Hence, we arrive at

$$z_{ss}(\theta) < z^*_{ss}(\theta) \iff -\frac{d}{dz}\left\{\lambda(z;\theta)\left(\mathbb{E}[V(z;\theta')|\theta] - V(z;\theta)\right)\right\}\Big|_{z=z_{ss}(\theta)} > 0. \tag{C.19}$$

Recall that in this model, the state variable represents the pollution stock, which should be reduced. This is why the first inequality in (C.19) is reversed and the minus sign is put on the left-hand side of the second inequality.

D Sensitivity to the assumption (not for publication)

We have assumed that there exists a level $\bar{z}$ of resource stock above which the hazard rate becomes zero. This section discusses how our results are changed when this assumption is relaxed. To this end, we consider a hazard rate function of the form

$$\lambda(z;\theta) = z^{-\omega}, \tag{D.1}$$

where ω captures the sensitivity of the hazard rate to z. Notice that this specification fails to satisfy Assumption 1 because the hazard rate is always positive.

Since the equilibrium is still characterized by (A.34) and (A.35), the equilibrium extraction rate is numerically solvable. We have tried various initial conditions and found that there exists a solution of the differential equation that converges to $\beta^*(\theta)$ as $z \to \infty$, which we denote by $\beta^l_\infty(z;\theta)$. Within the range of our numerical specification, it seems that this is the unique solution that satisfies the transversality condition as well. The trajectory of $\beta^l_\infty(z;\theta)$ in regime θ_1 is depicted in Figure 8. As is clear from the figure, we still observe the precautionary effect in a wide interval of the state space. In particular, the equilibrium extraction rate remains strictly below the benchmark level for all sufficiently large z.

When the sensitivity parameter ω of hazard rate function is sufficiently small, however, our results in the main text are not reproduced by (D.1). Figure 9 demonstrates this point. For this numerical example, we assume $\omega = 1$ and keep the other parameter values the same as in Figure 8. If we specify $\lambda(z;\theta)$ by (2.3), which satisfies Assumption 1, then there still exits a similar precautionary effect. If we replace $\lambda(z;\theta)$ by (D.1), which does not satisfy Assumption 1, then the equilibrium extraction rate $\beta^l_\infty(z;\theta)$ always remains above $\beta^*(\theta)$, meaning that there is no precautionary effect in this case. Hence, we conjecture that our results will remain valid without Assumption 1 if the curvature of the hazard rate function is sufficiently large.

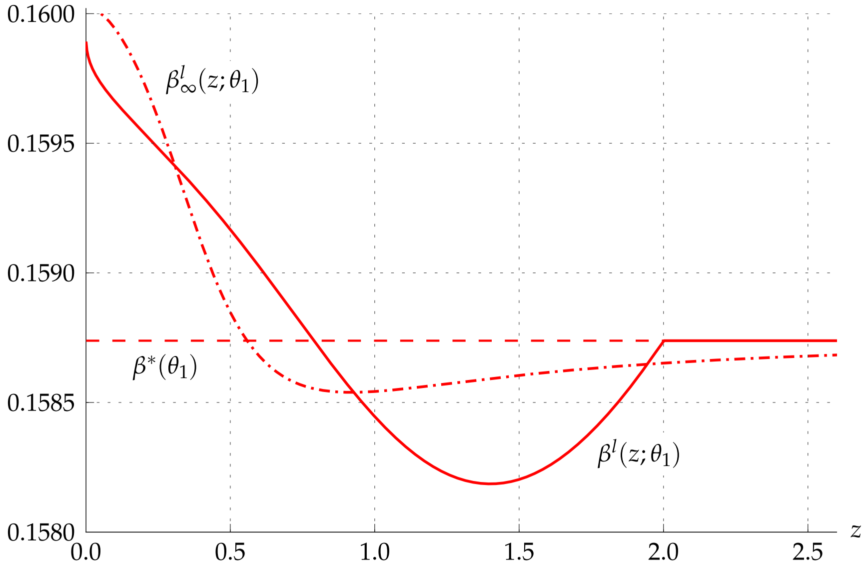

Figure 8: Equilibrium extraction rate in regime θ_1 with the same numerical specification as in Figure 2.

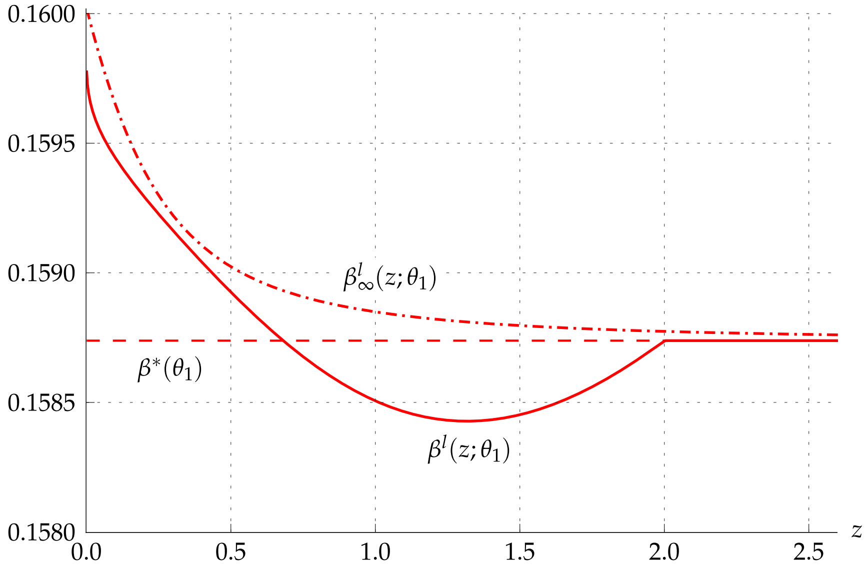

Figure 9: Equilibrium extraction rate in regime θ_1 with the same numerical specification as in Figure 2 except for $\omega = 1$.